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Artificial Equilibrium Points Maintained by an Electric Sail

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Abstract

The existence and stability of equilibrium points in the elliptic restricted three body problem for electric sails are investigated.

1. Introduction

An Electric Sail (ES) is an innovative propulsion system that uses charged tethers to extract momentum from the solar wind to produce a thrust without any ejection of mass [1]. Recently, the dynamical behavior of an ES within a two body problem has been investigated. Here we study the behavior of an ES in an elliptic restricted three body problem (ER3BP). We start with the dimensionless equation of motion in a synodic pulsating frame, then we show the existence of equilibrium points on the plane of motion of the two massive bodies and study the positions and the stability of some of these points.

2. Mathematical Model

Consider an ER3BP in which an infinitesimal object (the spacecraft) is subjected to a force provided by an ES and to the gravitational attraction due to the two attractors of mass m_1 and m_2, with $m_2 \leq m_1$.

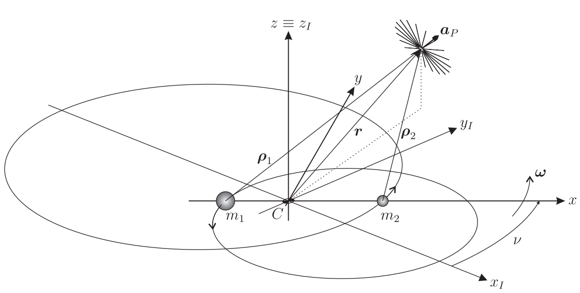

Figure 1: Electric Sail in the ER3BP.

Assume that the propulsive acceleration $\boldsymbol{a}_P$ relative to m_1 is purely radial and that it varies with the distance as $\rho_1^{-\eta}$, with $\eta = 7/6$ [1].

Introduce a dimensionless pulsating synodic frame $\mathcal{T}_p$ with axes parallel to the frame $\mathcal{T}(C;\ x,\ y,\ z)$, see Fig. 1. Using the transformation $\widetilde{\square} = \square/\rho$ [2], where ρ is the distance between the attractors, the equation of motion of the third body is:

$$\frac{\mathrm{d}^2\tilde{\boldsymbol{r}}}{\mathrm{d}\nu^2} + 2\hat{\boldsymbol{\omega}} \times \frac{\mathrm{d}\tilde{\boldsymbol{r}}}{\mathrm{d}\nu} = g\left[-\frac{1-\mu}{\tilde{\rho}_1^3}\tilde{\boldsymbol{\rho}}_1 - \frac{\mu}{\tilde{\rho}_2^3}\tilde{\boldsymbol{\rho}}_2 + \frac{\beta}{(\rho/a)^{\eta-2}}\frac{1-\mu}{\tilde{\rho}_1^{\eta+1}}\tilde{\boldsymbol{\rho}}_1 - \hat{\boldsymbol{\omega}} \times (\hat{\boldsymbol{\omega}} \times \tilde{\boldsymbol{r}}) - e\cos\nu\,\tilde{\boldsymbol{r}}_\parallel\right] \quad (1)$$

where ν, e, and a are the true anomaly, eccentricity, and semimajor axis of the orbit of m_2 with respect to m_1, $\hat{\boldsymbol{\omega}}$ is the angular velocity unit vector, $\tilde{\boldsymbol{r}}_\parallel$ is the component of $\tilde{\boldsymbol{r}}$ along $\boldsymbol{\omega}$, $\mu \triangleq m_2/(m_1 + m_2)$, β is the sail lightness number, that is, the ratio between the $\|\boldsymbol{a}_P\|$ and the gravitational acceleration due to m_1 at $\rho_1 = a$, and $g \triangleq 1/(1 + e\cos\nu)$. The equilibrium solutions are obtained by enforcing the conditions $\mathrm{d}^2\tilde{\boldsymbol{r}}/\mathrm{d}\nu^2 \equiv \mathrm{d}\tilde{\boldsymbol{r}}/\mathrm{d}\nu = 0$ into Eq. (1) [2]:

$$-\frac{1-\mu}{\tilde{\rho}_1^3}\tilde{\boldsymbol{\rho}}_1 - \frac{\mu}{\tilde{\rho}_2^3}\tilde{\boldsymbol{\rho}}_2 + \frac{\beta}{(\rho/a)^{\eta-2}}\frac{1-\mu}{\tilde{\rho}_1^{\eta+1}}\tilde{\boldsymbol{\rho}}_1 - \hat{\boldsymbol{\omega}} \times (\hat{\boldsymbol{\omega}} \times \tilde{\boldsymbol{r}}) - e\cos\nu\,\tilde{\boldsymbol{r}}_\parallel = 0\,. \quad (2)$$

It can be shown that, if $e \neq 0$, only solutions in the plane of motion ($z = 0$) of the attractors are possible, and that these solutions exist only if $B \triangleq \beta/(\rho/a)^{\eta-2}$ is constant. Focus now our attention on the equilibrium points (*triangular points*) that are not aligned with the attractors. From Eq. (2), the position of the triangular points (in pair, by symmetry), for a given value of B, is obtained from:

$$-\frac{1}{\tilde{\rho}_1^3} + 1 + \frac{B}{\tilde{\rho}_1^{\eta+1}} = 0 \quad , \quad \tilde{\rho}_2 = 1 \qquad (3)$$

Accordingly, the equilibrium points, see Fig. 2, are placed on the circumference centered at m_2 and with unit radius. An increase of B brings the point closer to m_1, independently of μ. For example, an equilibrium point at a distance $\tilde{\rho}_1 \approx 0.983$ from m_1 requires $B = 0.05$, meaning that β must vary as per Fig. 3, if one considers the Sun-[Earth+Moon] system. If one introduces a perturbation of position $\delta\tilde{\boldsymbol{r}}$ and velocity

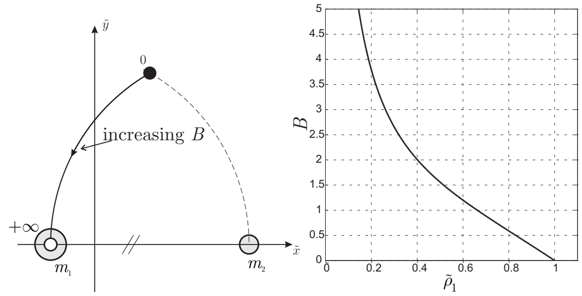

Figure 2: Locus of the triangular points and their positions as a function of parameter B.

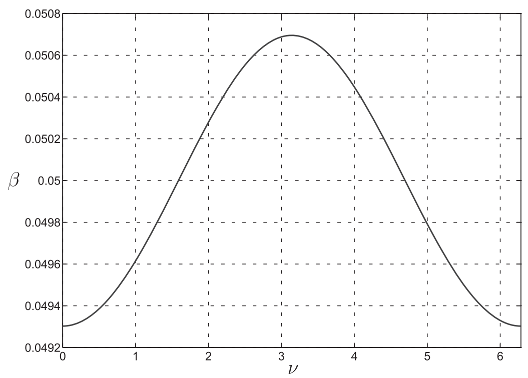

Figure 3: Required variation of β to maintain a triangular point at $\tilde{\rho}_1 = 0.983$ in the Sun-[Earth+Moon] system.

$\delta\tilde{\boldsymbol{v}} = \mathrm{d}(\delta\tilde{\boldsymbol{r}})/\mathrm{d}\nu$, Eq. (1) can be linearized in a neighborhood of an equilibrium point to check its stability. The linearized equation can be written as a first-order differential equation system in the form

$$\left[\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\nu}\right]_{\mathcal{T}_p} = \mathbb{M}\,[\boldsymbol{x}]_{\mathcal{T}_p} \qquad (4)$$

where $\boldsymbol{x} = [\delta\tilde{\boldsymbol{r}},\, \delta\tilde{\boldsymbol{v}}]^{\mathrm{T}}$ and $\mathbb{M} \in \mathbb{R}^{6\times 6}$ is a ν-dependent matrix.

Because the $\mathbb{M}$ entries are 2π-periodic, the stability of (4) depends (by Floquet Theory) on the eigenvalues λ_i of the monodromy matrix $\mathbb{C} = \mathbb{F}(2\pi)$, where $\mathbb{F}(\nu)$ is the solution of $\dot{\mathbb{F}}(\nu) = \mathbb{M}(\nu)\mathbb{F}(\nu)$ with $\mathbb{F}(0) = \mathbb{I}$. If $|\lambda_i| \leq 1$ the system is stable. The latter condition has been investigated via numerical integration. Assuming $\tilde{\rho}_1 = 0.983$, the stability of triangular points is guaranteed for each pair of μ and e in the gray regions of Fig. 4. For a given pair of μ and e, e.g. the Sun-[Earth+Moon] system, all of the triangular equilibrium points in Fig. 5 are stable.

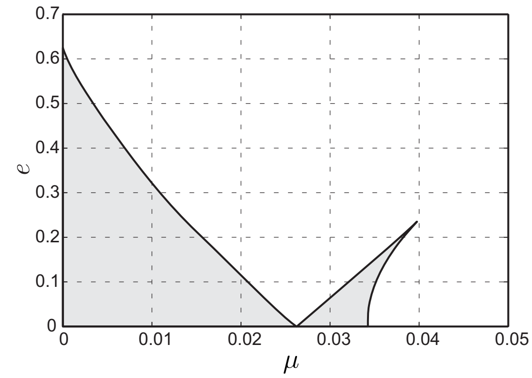

Figure 4: Regions of stability of the triangular points at $\tilde{\rho}_1 = 0.983$ in the $\mu - e$ plane.

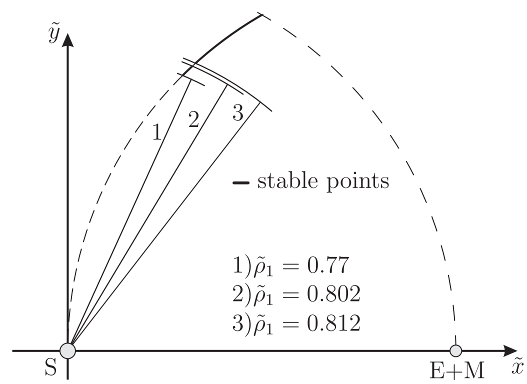

Figure 5: Stable triangular points in the Sun-[Earth+Moon] system.

3. Conclusions

The concept of generating equilibrium points in the ER3BP by means of an ES has been developed. A variable thrust, provided by the ES, is required to obtain equilibrium points in a synodic pulsating frame. These points are located on the plane of motion of the two massive bodies.

References

[1] Janhunen, P. and Sandroos, A., “Simulation study of solar wind push on a charged wire: basis of solar wind electric sail propulsion,” *Annales Geophysicae*, Vol. 25, No. 3, 2007, pp. 755–767. doi: 10.1029/2006GL028602.

[2] Szebehely, V., *Theory of orbits: the restricted problem of three bodies*, Academic Press Inc., 1967, pp. 587–599.