

Equilibrium shapes of rubble pile asteroids

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Abstract

A significant fraction of asteroids are probably aggregates of boulders kept together by simple gravity. If these bodies were self-gravitating and rotating cohesionless fluids, they would be found close to one of the well-known equilibrium sequences of MacLaurin and Jacobi; conversely, it is known this not being the case. This discrepancy, although expected on the basis of their rocky nature, is surprisingly small when quantitatively evaluated. In this work an analysis of the preferential shapes a gravitational aggregate tends to assume under the action of self-gravity is presented: the results show that actual asteroid shapes are consistent with evolution of aggregates of free components tending towards minimum free energy states.

1. Introduction

Current models suggest that many medium-to-large asteroids are not monolithic in nature, but rather “rubble piles” of fragments sticking together by simple gravity. This conclusion is mainly supported by a series of facts: the presence of “asteroid families” of asteroids of similar orbits and composition suggesting a shattered parent body [Michel et al.(2001)]; the diameter distribution of asteroids $dn \propto D^{-2.5}dD$ compatible with the equilibrium for a collisional population of frequently mutually shattering bodies [Dohnanyi(1969)]; the limit of $\approx 2h$ of the rotation period of asteroids larger than $\sim 100m$ corresponding to cohesionless aggregates on the threshold of mass shedding [Harris(2002)]; the low bulk densities of many asteroids with respect to meteorite rocks of similar spectroscopic properties suggesting internal voids [Britt et al.(2002)] consistent with a fragmented structure.

Due to the rocky nature of the components, the classic hydrodynamic theory of equilibrium of fluid bodies [Chandrasekhar(1969)] does not seem apt to directly describe the general trend of actual shapes. It is not surprising that the observed shapes don’t cluster around the MacLaurin and Jacobi ellipsoidal se-

quences, as illustrated in fig. 1. It is clear that the observed axis ratios scatter on a wide range of values, apparently unrelated to fluid equilibrium. On the other hand, even complex shapes such as those of 216 Kleopatra or 25143 Itokawa exhibit small local slopes relative to the local potential surface (less than ~ 10 degrees), with asteroids with satellite seeming even closer [Hestroffer and Tanga(2005)].

In the following we expand on the results obtained in [Tanga et al. (2009)], showing that the classic fluid equilibrium shapes constitute an attractor for the evolution of ellipsoids of arbitrary initial axis ratios.

2. The model

For an aggregate body in rigid rotation, its mechanical energy will be the sum of the gravitational energy U and of the rigid body rotational energy E_r . A certain rigid body shape for a given spin can be maintained only if the object is at equilibrium, otherwise beginning a process of reshaping with loss of energy due to internal frictions before eventually come to a rigid body rotation configuration. As this process does conserve angular momentum \vec{L} , and supposing constant density, it allows us to define a normalised quantity $\tilde{E}_L = \tilde{U} + \tilde{E}_r$ depending only on the shape of the body, and to assume that reshaping shall occur in the direction minimizing such quantity, i.e. following $-\vec{\nabla}_{shapespace}\tilde{E}_L$.

Assuming an ellipsoidal shape with semiaxes a, b, c (c being parallel to \vec{L}), in the classical incompressible fluid case, a self-gravitating body at equilibrium accommodates to an equipotential surface, always orthogonal to the local (gravitational + centrifugal) force, resulting in the MacLaurin and Jacobi sequences. This model assumes an absence of resistance to internal reshaping, which is not the case for real rocky asteroids.

If a body is free to evolve from a starting non-equilibrium configuration in rigid rotation, it will migrate on the $(b/a); (c/a)$ plane of constant \vec{L} along the direction of $-\vec{\nabla}_{b/a;c/a}\tilde{E}_L$ towards the minima of the MacLaurin or the Jacobi shapes. In an area around such ideal minima, however, the thrust for reshap-

ing is so low to no longer being able to overcome the tiny friction presented by the body: the process of reshaping thus may come to a halt. The resulting final states were obtained via numerical simulations with the PKDGRAV code [Richardson(1993)] on bodies composed of identical spheres.

3. Conclusions

Comparing (fig. 1) the diagram of final shapes for multiple values of Angular Momentum to the distribution of real asteroids, the similarity is obvious [Tanga et al. (2009)].

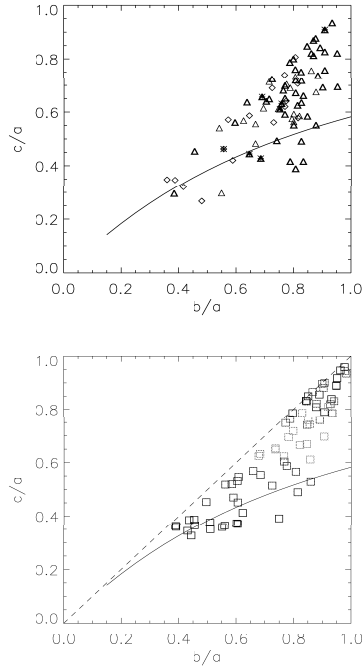


Figure 1: Top panel: Shapes of asteroids for which both axis ratios are known; bottom: Final shapes for simulations using $\bar{L} = 0 ; 0.1 ; 0.2 ; 0.3 ; 0.4 ; 0.5$ [This \bar{L} normalization is so that 1 corresponds to an omogeneous sphere on the verge of centrifugal mass shedding] and a variety of initial shapes. The Jacobi sequence is shown with continuous line, MacLaurin sequence coinciding with the right side of the box.

Actual shapes appear perfectly compatible with a “rubble pile” gravitational aggregate with even a mildly departure (5 degree slope) from a fluid system,

arguably less than the repose angle for a pile of rocky or icy boulders. This result does not imply that the constituting material should have this low angle of repose, but simply that the history of the body, by the progressive gradual action of small stresses (due to minor impacts, tidal forces...) allows it to flow gradually toward a more fluid-like final state.

One should note that our conclusions are based on the analysis of known axis ratios only. This can be seen as a limitation, but our approach avoids poorly known parameters (such as the density) that could seriously affect the conclusions reached in previous papers [Holsapple(2004)]. Further observational data [Torppa et al. (2008)] not considered in [Tanga et al. (2009)] appear to be consistent with the proposed model and further support our evolution scenario for the origin of the observed shapes.

References

- [Britt et al.(2002)] Britt, D.T. et al., 2002. Asteroid Density, Porosity, and Structure. *Asteroids III*, p.485.
- [Chandrasekhar(1969)] S. Chandrasekhar, *Ellipsoidal figures of equilibrium*, New Haven and London, Yale University Press, 1969.
- [Dohnanyi(1969)] J.W. Dohnanyi, *J. Geophys. Res.* 74 (1969), 2531.
- [Harris(2002)] Harris, A.W., On the slow rotation of asteroids. *Icarus* 156 (2002), 184.
- [Hestroffer and Tanga(2005)] Hestroffer, D. and P. Tanga 2005. Figures of Equilibrium among Binary Asteroids. *BAAS* 37, 1562.
- [Holsapple(2001)] K.A. Holsapple, *Icarus* 154 (2001), 432.
- [Holsapple(2004)] K.A. Holsapple, *Icarus* 172 (2004), 272.
- [Lai et al.(1993)] D. Lai, F.A. Rasio, S.L. Shapiro, *The Astrophysical Journal Supplement Series* 88 (1993), 205.
- [Michel et al.(2001)] P. Michel, W. Benz, P. Tanga, D.C. Richardson, *Science* 294 (2001), 1696.
- [Richardson(1993)] D.C. Richardson, *Mon.Not.R.Astron.Soc.* 261 (1993), 396.
- [Tanga et al. (2009)] Tanga, P., C. Comito, P. Paolicchi, D. Hestroffer, A. Cellino, A. Dell’Oro, D. C. Richardson, K. J. Walsh, and M. Delbo 2009. *ApJ lett.* 706, L197.
- [Torppa et al. (2008)] Torppa, J., Hentunen, V.P., Paakkonen, P., Kehusmaa, P., Muinonen, K., Asteroid shape and spin statistics from convex models, *Icarus* 198, 1, 91.