

# Measurement Requirements for Quantifying Titan's Air-Sea Interactions

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## Abstract

A major goal of a Titan mission concept study commissioned under the recent Decadal Survey was to study lake-atmosphere interactions in order to determine the role of Titan's lakes in the methane cycle. Achieving this goal requires measuring the appropriate parameters in order to quantify the exchange of mass, energy, and momentum between the atmosphere and an underlying liquid reservoir. This paper provides traceability from lake-atmosphere interaction theory to measurements.

## 1. Radiative and Convective Fluxes

Energy exchange may occur in one or more ways: radiation, conduction, or convection. The magnitude of radiation exchange can be estimated by considering the temperature derivative of the Stefan-Boltzmann Blackbody Law:

$$\frac{dE}{dT} = 4\sigma T^3. \quad (1)$$

For a surface temperature of 94 K, this corresponds to  $\sim 0.2 \text{ W m}^{-2} \text{ K}^{-1}$ . If the differential temperature between atmosphere and lake is  $\sim 5 \text{ K}$ ,  $\sim 1 \text{ W m}^{-2}$  in radiative heat exchange is expected. Although not a large value, this is of the same order as the solar input at the surface. In the absence of other energy exchanges or over seasonal timescales, the measurement of radiation flux exchanges would be important to understanding the atmosphere-lake energy balance. Further, if the energy balance is to be measured, the accuracy/resolution should be approximately an order of magnitude smaller than the expected range:  $\sim 0.02 \text{ W m}^{-2}$  in the infrared. For approximately  $1 \text{ W m}^{-2}$  of solar input (some direct and mostly diffuse), an accuracy/resolution of  $\sim 0.1 \text{ W m}^{-2}$  is required in the broadband visible.

Exchange via conduction and convection is generally expressed as a flux of energy or mass. Simple scaling arguments show that convection will dominate over conduction by many order of magnitudes. Therefore, turbulent eddies are the remaining mechanism by which air-lake interactions occur.

The magnitude of the turbulent eddy contribution to momentum and scalars (including heat and

methane) is obtained by computing the eddy flux convergence:

$$\left( \frac{\partial \bar{u}_i}{\partial t} \right)_{turb} = - \frac{\partial \langle u_i u_j' \rangle}{\partial x_j}, \quad (2)$$

$$\left( \frac{\partial \bar{\chi}}{\partial t} \right)_{turb} = - \frac{\partial \langle \chi' u_j' \rangle}{\partial x_j}. \quad (3)$$

where overbars indicate time-mean fields and primes indicate time perturbations from the mean.

Eqs. (2)-(3) clearly indicate that the three-dimensional gradient of the turbulent fluxes are required to obtain the total eddy contribution. However, because the exchange between the lake and the atmosphere is in the vertical direction, the direct turbulent contribution from the lake-atmosphere contribution is obtained by neglecting horizontal gradients. Therefore, the relevant eddy flux contribution requires the measurement of the fluxes at a minimum of two heights in order to obtain a vertical gradient. A minimum of three measurement heights are required to obtain a solution for the fluxes at the surface rather than at the measurement height.

## 2. Closure of Turbulent Fluxes

Measurement of the vertical gradient of fluxes is sufficient to determine the magnitude of the turbulent tendency at the measurement heights. What the measurement does not provide is any indication of how the fluxes are related to the overall mean lake and atmosphere properties or what the flux is at the atmosphere-lake interface, but this information is necessary to extend the measurements globally.

The vertical structure of turbulent fluxes is rooted in Buckingham II theorem [1] under the assumption that there exists a constant stress layer in the atmosphere. Turbulence is taken to be governed by four key parameters [2]:

- i) the friction velocity  $u_* = (\tau/\rho)^{1/2}$ , where  $\tau$  is the (constant) surface stress and  $\rho$  is the atmospheric density; ii) a non-dimensional surface temperature scale,  $\theta_* = Q_o/u_*$ , where the surface heat flux  $Q_o = (w' \theta'_o)$  given as the covariance between vertical velocity ( $w$ ) and temperature ( $\theta$ ); iii) the Obukhov Length  $L = -\frac{u_*^3 T}{k g Q_o}$ , where  $k$  is Von

Karman's constant and  $g$  is gravitational acceleration; and iv) the physical height  $z$  above the surface.  $L$  is the only dimensional length scale that can be formed from the two parameters above.  $L > 0$  when the heat flux is negative (stable),  $L < 0$  when the heat flux is positive and the atmosphere is buoyantly convecting (unstable). Using these four parameters dimensional analysis dictates that surface layer flow properties may be expressed by universal functions with an argument  $\zeta = z/L$ :

$$\frac{kz}{u_*} \frac{\partial \bar{u}}{\partial z} = \phi_m(\zeta), \quad (4)$$

$$\frac{kz}{\theta_*} \frac{\partial \bar{\theta}}{\partial z} = \phi_h(\zeta), \quad (5)$$

$$\frac{kz}{q_*} \frac{\partial \bar{q}}{\partial z} = \phi_h(\zeta). \quad (6)$$

If the so-called structure functions are known, it becomes possible to determine the characteristic surface layer wind, potential temperature, and moisture values given the mean values. The structure functions may be considered as non-dimensionalized shear (for momentum) and buoyancy (for  $\theta$ ) parameters. The ratio of the two functions is the Richardson Number.

Integrating Eqs. (4)-(6) from  $z=z_o$  to  $z=z_a$ , where  $a$  refers to the anemometer height, results in:

$$\rho_o \langle \bar{w}' \bar{\theta}' \rangle_o = \rho_o \left[ \frac{k^2}{\left( \int_{z_o}^{z_a} \frac{1}{z} \phi_m(\xi) dz \right)^2} \right] u_a (\theta_a - \theta_o), \quad (7)$$

$$\rho_o \langle \bar{w}' \bar{\theta}' \rangle_o = \rho_o \left[ \frac{k^2}{\left( \int_{z_o}^{z_a} \frac{1}{z} \phi_m(\xi) dz \int_{z_o}^{z_a} \frac{1}{z} \phi_h(\xi) dz \right)^2} \right] u_a (\theta_a - \theta_o), \quad (8)$$

$$\rho_o \langle \bar{w}' \bar{q}' \rangle_o = \rho_o \left[ \frac{k^2}{\left( \int_{z_o}^{z_a} \frac{1}{z} \phi_m(\xi) dz \int_{z_o}^{z_a} \frac{1}{z} \phi_h(\xi) dz \right)^2} \right] u_a (q_a - q_o). \quad (9)$$

The terms in brackets in Eqs. (7)-(9) are the bulk transfer coefficients. Thus, once the structure functions are known as a function of height and  $\zeta$ , it is possible to determine the eddy fluxes at the surface  $z=z_o$ , which is ultimately what is desired. Assuming that the structure functions vary smoothly with limited changes of inflection, a minimum of three points are needed to estimate the shape of the structure functions sufficient for calculation of the integral. Additional points are beneficial.

Based on the arguments above, the list of parameters in Table 1 are required to satisfy the science goal of air-lake interactions. Also listed in the table are sampling rates, which are justified later. The strategy is to measure the mean properties and eddy fluxes at a minimum of three heights, plus the lake temperature and composition. These measurements allow for the empirical determination

of the structure functions (Eqs. (4)-(6)) and for the integration of the functions to obtain the bulk transfer coefficients and the value of the surface fluxes (Eqs. (7)-(9)).

**Table 1. Key Measurements**

Parameter	Use
Surface Lake Temperature, $T_o$	Calculate $\text{IR} \uparrow$ ; saturation vapor pressure; Surface heat flux.
Pressure*, $p$	Potential temperature, $\theta(T,p)$
$u^*, v^*, w^*$	3-D mean wind for structure functions and eddy covariances.
Atmos. Temp., $T^*$	$\theta(T,p)$ for structure function and bulk heat flux calculation.
Volatile abundance*	structure functions, $\phi_m$ , $\phi_h$ , and bulk volatile flux calculation
Lake Composition	Establish saturation vapor pressure of $\text{C}_x\text{H}_y$ .
Solar Flux, $F \downarrow$	Radiative heating
IR $\downarrow$	Radiative Heating
Rain rate <sup>+</sup>	Mass return to lake

\*Taken at a minimum of three heights (e.g., 0.3 m, 0.6 m, 1.0 m);<sup>+</sup>Not a floor requirement, but technically part of lake-atmosphere surface budget.

Parameters required for eddy flux determination must be sampled at high enough frequency to capture the structure of the eddies, and over a long enough time for a statistically significant sampling of many eddies. The size of eddies scale roughly with the height above the surface. For a measurement at 0.3 m AGL, an eddy moving with the mean wind of 0.1 m/s would pass the measurement station in 3 s. To adequately resolve the structure of the eddy requires roughly a decade of samples or 1 sample per 0.3 s. For a 1 m/s wind, the sampling is reduced to 1 per 0.03 s. A rate of  $\sim$ 10 Hz is therefore acceptable.

Solar and IR fluxes are not expected to vary substantially over short timescales; the dominate scales are diurnal and seasonal. Sampling  $\sim$ 1 hr $^{-1}$  or even less frequently is likely sufficient to capture the relevant changes. Due to the large thermal mass of the liquid reservoirs, rapid variations in temperature are not anticipated, but ensemble eddy exchanges may force detectable changes thus requiring lake temperature sampling of  $\sim$ 0.1 Hz.

## References

- [1] Buckingham, E.: On physically similar systems; illustrations of the use of dimensional equations, *Phys. Rev.* **4**: 345–376. doi:10.1103/PhysRev.4.345, 1914.
- [2] Monin, A.S., A. M. Obukhov: Basic laws of turbulent mixing in the surface layer of the atmosphere, *Tr. Geofiz. Inst. Akad. Nauk, SSSR*, **151**, 163–187, 1954.