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LLR and the Lense-Thirring Effect

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Abstract

We mainly explore the possibility of measuring the action of the intrinsic gravitomagnetic field of the rotating Earth on the orbital motion of the Moon with the LLR technique. Expected improvements in it should push the precision in measuring the Earth-Moon range to the mm level; the present-day rms accuracy in reconstructing the radial component of the lunar orbit is about 2 cm; its harmonic terms can be determined at the mm level. The current uncertainty in measuring the lunar precession rates is about 10^{-1} milliarcseconds per year. The Lense-Thirring secular precessions of the node and the perigee of the Moon induced by the Earth's spin angular momentum amount to 10^{-3} milliarcseconds per year yielding transverse and normal shifts of $10^{-1} - 10^{-2}$ cm yr^{-1}. In the radial direction there is only a short-period, i.e. non-averaged over one orbital revolution, oscillation with an amplitude of 10^{-5} m. Major limitations come also from some systematic errors induced by orbital perturbations of classical origin like, e.g., the secular precessions induced by the Sun and the oblateness of the Moon whose mismodelled parts are several times larger than the Lense-Thirring signal. The present analysis holds also for the Lue-Starkman perigee precession due to the multidimensional braneworld model by Dvali, Gabadadze and Porrati (DGP); indeed, it amounts to about 5×10^{-3} milliarcseconds per year.

1 Introduction

We consider in some details the possibility of measuring with LLR a general relativistic effect induced by the gravitomagnetic field of the spinning Earth through a non-central, Lorentz-like acceleration on the orbital motion of the Moon around the Earth, i.e. the Lense-Thirring precessions of the longitude of the ascending node Ω and the argument of pericentre ω.

Our analysis is equally valid also for the anomalous Lue-Starkman perigee precession predicted in the framework of the multi-dimensional braneworld model of modified gravity put forth by Dvali, Gabadadze and Porrati (DGP) to explain the observed acceleration of the Universe without resorting to dark energy; indeed, as we will see, the magnitude of such an effect is the same as the Lense-Thirring one for the Moon. Several researchers argued that it might be possible to measure the Lue-Starkman precession with LLR in view of the expected improvements in such a technique.

2 The Lense-Thirring effect on the lunar orbit and some sources of error

By assuming a suitably constructed geocentric equatorial frame, it turns out that the node and the perigee of the Moon undergo the Lense-Thirring secular precessions of 0.001 mas yr^{-1} and -0.003 mas yr^{-1}, respectively. The Lue-Starkman LS pericentre precession is just about ∓ 0.005 mas yr^{-1}.

Let us now examine some sources os systematic errors. In regard to the potentially corrupting action of the mismodelling in the even ($\ell = 2, 4, 6, ...$) zonal ($m = 0$) harmonic coefficients J_ℓ of the multipolar expansion of the Newtonian part of the Earth's gravitational potential, which is not the most important source of aliasing precessions in the case of the Moon, only δJ_2 would be of some concern. Indeed, the mismodelled secular precessions induced by it on the lunar node and perigee amount to -2.67×10^{-4} mas yr^{-1} and 5.3×10^{-4} mas yr^{-1}, respectively; the impact of the other higher degree even zonals is negligible being $\leq 10^{-8}$ mas yr^{-1}. As in the case of the spins, also the asphericity of the Moon has to be taken into account according to

$$\dot{\Omega}_{J_2^{\rm Moon}} = -\frac{3}{2}\frac{n_{\rm Moon}\cos F J_2^{\rm Moon}}{(1-e^2)^2}\left(\frac{R_{\rm Moon}}{a}\right)^2, \quad (1)$$

where $n_{\rm Moon} = \sqrt{GM(1+\mu)/a^3}$ is the lunar mean motion, μ is the Moon-Earth mass ratio, and F is the angle between the orbital angular momentum and the Moon's spin angular momentum $\boldsymbol{S}_{\rm Moon}$; it is about

3.61 deg since the spin axis of the Moon is tilted by 1.54 deg to the ecliptic and the orbital plane has an inclination of 5.15 deg to the ecliptic. The mismodelled node precession due to $\delta J_2^{\rm Moon}$ is about 0.006 mas yr^{-1}, i.e. 6 times larger than the Lense-Thirring rate. Among the N-body gravitational perturbations, the largest ones are due to the Sun's attraction. In order to get an order-of-magnitude evaluation of their mismodelling, let us note that some of such effects are proportional to $n_{\oplus}^2/n_{\rm Moon}$; e.g. the node rate, referred to the equator, is

$$\dot\Omega^{\odot} = \frac{3GM_{\odot}\cos I}{4a_{\oplus}^3 n_{\rm Moon}}\left(\frac{3}{2}\sin^2\varepsilon - 1\right) \approx -5\times 10^7\ \frac{\rm mas}{\rm yr}, \quad (2)$$

where $\varepsilon = 23.439$ deg is the obliquity of the ecliptic. Since $\delta GM_{\odot} = 5\times 10^{10}$ m^3 s^{-2} and $\delta GM = 8\times 10^5$ m^3 s^{-2}, we can assume a bias of $\approx$ 0.07 mas yr^{-1} which is 70 times larger than the Lense-Thirring precession.

Let us, now, consider the precision of LLR in reconstructing the lunar orbit with respect to the Lense-Thirring effect. Concerning the precision in measuring the lunar precession rates, it amounts to about 0.1 mas yr^{-1}, i.e. it is two orders of magnitude larger than the Lense-Thirring precessions. The orbital perturbations experienced by a test particle are usually decomposed along three orthogonal directions of a frame co-moving with it; they are named radial R (along the radius vector), transverse T (orthogonal to the radius vector, in the osculating orbital plane) and normal N (along the orbital angular momentum, out of the osculating orbital plane). The $R-T-N$ perturbations can be expressed in terms of the shifts in the Keplerian orbital elements. The lunar Lense-Thirring shifts after one year are, thus, about -0.40 cm and 0.07 cm in the T and N directions, respectively. It is important to note that there is no Lense-Thirring secular signature in the Earth-Moon radial motion on which all of the efforts of LLR community have been concentrated so far. It can be shown that a short-period, i.e. not averaged over one orbital revolution, radial signal exists; it is proportional to

$$\Delta r \propto \frac{2GS_{\oplus}}{c^2 n a^2} = 2\times 10^{-5}\ {\rm m}, \quad (3)$$

which is too small to be detected since the present-day accuracy in estimating the amplitudes of radial harmonic signals is of the order of mm. Major limitations come from the post-fit rms accuracy with which the lunar orbit can be reconstructed; the present-day accuracy is about 2 cm in the radial direction R along the centers-of-mass of the Earth and the Moon. Improvements in the precision of the Earth-Moon ranging of the order of 1 mm are expected in the near future with the APOLLO program. Recently, sub-centimeter precision in determining range distances between a laser on the Earth's surface and a retro-reflector on the Moon has been achieved. However, it must be considered that the rms accuracy in the T and N directions is likely worse than in R.

3 Conclusions

We have examined the possibility of measuring the action of the intrinsic gravitomagnetic field of the spinning Earth on the lunar orbital motion with the LLR technique. After showing that the Lense-Thirring approximation is adequate for the Earth-Moon system, we found that the Lense-Thirring secular precessions of the Moon's node and the perigee induced by the Earth's spin angular momentum are of the order of 10^{-3} mas yr^{-1} corresponding to transverse and normal secular shifts of $10^{-1}-10^{-2}$ cm yr^{-1}. The intrinsic gravitomagnetic field of the Earth does not secularly affect the radial component of the Moon's orbit; a short-period, i.e. not averaged over one orbital revolution, radial oscillation is present, but its amplitude is of the order of 10^{-5} m. The current rms accuracy in reconstructing the lunar orbit is of the order of cm in the radial direction; the harmonic components can be determined at the mm level. Forthcoming expected improvements in LLR should allow to reach the mm precision in the Earth-Moon ranging. The present-day accuracy in measuring the lunar precessional rate is of the order of 10^{-1} mas yr^{-1}. Major limitations come also from some orbital perturbations of classical origin like, e.g., the secular node precessions induced by the Sun and the oblateness of the Moon which act as systematic errors and whose mismodelled parts are up to 70 times larger than the Lense-Thirring effects. As a consequence of our analysis, we are skeptical concerning the possibility of measuring intrinsic gravitomagnetism with LLR in a foreseeable future. The same conclusion holds also for the Lue-Starkman perigee precession predicted in the framework of the multidimensional braneworld DGP model of modified gravity; indeed, it is as large as the Lense-Thirring one for the Moon.