



# Something on the LAGEOS-based Lense-Thirring tests

**L. Iorio**

F.R.A.S. Viale Unità di Italia 68, 70125, Bari (BA) Italy (lorenzo.iorio@libero.it)

## Abstract

The strategy followed so far in the performed or proposed tests of the general relativistic Lense-Thirring effect in the gravitational field of the Earth with laser-tracked satellites of LAGEOS type relies upon the cancelation of the disturbing huge precessions induced by the first even zonal harmonic coefficient  $J_2$  of the multipolar expansion of the Newtonian part of the terrestrial gravitational potential by means of suitably designed linear combinations of the nodes  $\Omega$  of more than one spacecraft. Actually, such a removal does depend on the accuracy with which the coefficients of the combinations adopted can be realistically known. Uncertainties of the order of 2 cm in the semimajor axes  $a$  and 0.5 milliarcseconds in the inclinations  $I$  of LAGEOS and LAGEOS II, entering the expression of the coefficient  $c_1$  of the combination of their nodes used so far, yield an uncertainty  $\delta c_1 = 1.30 \times 10^{-8}$ . It gives an imperfectly canceled  $J_2$  signal of 10.8 milliarcseconds per year corresponding to 23% of the Lense-Thirring signature. Uncertainties of the order of 10 – 30 microarcseconds in the inclinations yield  $\delta c_1 = 7.9 \times 10^{-9}$  which corresponds to an uncanceled  $J_2$  signature of 6.5 milliarcseconds per year, i.e. 14% of the Lense-Thirring signal. Concerning a future LAGEOS-LAGEOS II-LARES combination with coefficients  $k_1$  and  $k_2$ , the same uncertainties in  $a$  and the less accurate uncertainties in  $I$  as before yield  $\delta k_1 = 1.1 \times 10^{-8}$ ,  $\delta k_2 = 2 \times 10^{-9}$ ; they imply a residual  $J_2$  combined precession of 14.7 milliarcseconds per year corresponding to 29% of the Lense-Thirring trend. Uncertainties in the inclinations at  $\approx 10$  microarcseconds level give  $\delta k_1 = 5 \times 10^{-9}$ ,  $\delta k_2 = 2 \times 10^{-9}$ ; the uncanceled  $J_2$  effect is 7.9 milliarcseconds per year, i.e. 16% of the relativistic effect.

## 1 The issue of the accuracy in the removal of Earth's first even zonal harmonic

One of the major sources of systematic uncertainty in the measurement of the gravitomagnetic Lense-Thirring precessions of the nodes  $\Omega$  of the laser-tracked LAGEOS-type satellites in the gravitational field of the Earth is given by the much larger competing classical node precessions induced by the even zonal harmonic coefficients  $J_\ell$ ,  $\ell = 2, 4, 6, \dots$  of the expansion in multipoles of the non-spherically symmetric terrestrial gravitational potential. The strategy followed so far to partially circumvent such an issue consisted of suitably designing linear combinations of the nodes of more than one satellite to cancel out, by construction, the effects of  $J_2$ , as in the ongoing LAGEOS-LAGEOS II test, and of  $J_4$  as well, as in the future LAGEOS-LAGEOS II-LARES scenario. In addition to the usual systematic uncertainty due to the mismodeling in the even zonals of higher degree which fully impact such combinations, another source of non-negligible uncertainty of gravitational origin has to be taken into account. It is due to the imperfect cancelation of the effects of  $J_2$  because of the uncertainty in the coefficients entering the combinations set up just to remove it. Indeed, the numerical values of such coefficients, released with just a few decimal digits so far, explicitly depend on the numerical values of the semimajor axes  $a$ , the inclinations  $I$  and the eccentricities  $e$  of the satellites involved. Thus, the uncertainties with which such Keplerian orbital elements are known unavoidably have repercussions onto the coefficients themselves. For uncertainties in the semimajor axes of 1 – 2 cm and of about 0.5 – 0.01/0.03 milliarcseconds in the inclinations we show that the resulting systematic bias due to the imperfect removal of the  $J_2$  signal may be as large as 14 – 29% of the Lense-Thirring signatures.