EPSC Abstracts
Vol. 5, EPSC2010-626, 2010
European Planetary Science Congress 2010

# Relativistic effects in the BepiColombo mission

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## 1. Introduction

Due to considerable increase of the accuracy level in modern space missions in the recent years, or expected in close-future missions, relativistic gravitational effects must be considered when computing spacecraft ephemerides and observables. Since General Relativity is invariant under coordinate transformations, we must consider the observables, i.e.: Range and Doppler signals modeled completely within general relativity.

In this poster, we present a software that integrates the equations of motion and determine the Doppler/Range signal directly from the space-time metric. This means that all the relativistic effects are taken into account. This software is based upon the integration of the geodesic equation [1] and the computation of time transfer thanks to Synge World function formalism [2, 3]. The distinction between proper time and coordinate time is also clearly made in order to have a consistent definition of the observables.

## 2. Simulated observables from the space-time metric

Our software directly generates Doppler/Range signal from the metric considered in a coherent way. The equations of motion are derived from this metric. The behavior of clock (namely the difference between proper time and coordinate time) is also modeled. And finally, the time transfer is also determined directly from the metric. This approach has the main advantage that, by definition, no relativistic term is forgotten. Moreover, it also allows to change the metric in order to see how the signal changes if we assume another theory of gravity than General Relativity.

### 2.1. Equations of motion

The equations of motion are directly derived from the metric thanks to the geodesic equations [1] integrated with respect to coordinate time.

$$\frac{d^2x^i}{dt^2} = -\Gamma^i_{00} - 2\Gamma^i_{0j}v_j - \Gamma^i_{jk}v_jv_k \qquad (1)$$

$$+\Gamma^0_{00}v_i + 2\Gamma^0_{0j}v_iv_j + \Gamma^0_{jk}v_iv_jv_k$$

where $x^i$ are the spatial coordinates of the spacecraft, $\Gamma^\alpha_{\beta\gamma}$ are the Christoffel symbols of the considered metric (Greck indices run from 0 to 3 while Latin indices from 1 to 3) and $t$ is coordinate time.

### 2.2. Clock behavior

In order to make a clear distinction between proper time and coordinate time, proper time equations are also integrated for each body considered, and in particular for the clocks used in the Doppler/Range measurements. This equation is

$$\frac{d\tau}{dt} = \sqrt{g_{00} - 2g_{0i}v_i - g_{ij}v_iv_j} \qquad (2)$$

where $g_{\mu\nu}$ is the space-time metric and $\tau$ the proper time.

### 2.3. Light propagation

Finally, the signal propagation has to be modeled. This has been done thanks to the Synge World function formalism. Within this formalism, one does not need to integrate the photon trajectory in order to get the time transfer or the frequency shift. But, these quantities can be expressed as integral of some function defined from the metric (and it's derivative) along the Minkowski path of the photon [2, 3].

### 2.4. Observables

The Range observable can be modeled as the difference between the receiver proper time (at reception) and the emitter proper time (at emission). In the same way, the Doppler observable is modeled as the ratio between the received signal frequency over the emitted frequency.

## 3. Simulations for the Bepicolombo mission

Bepicolombo is a space mission planned to be launched in 2014 to Mercury [4]. The main goals of this mission are the study of the origin of Mercury, its internal structure, its form, its composition,… Another goal of this mission is to test Einstein's theory of gravitation [5]. We simulate Doppler/Range signal for this mission in General Relativity with our software. This allows us to quantify very precisely the relativistic effects. For example, Fig 1 shows the relativistic effect on the Range signal for the BepiColombo mission while Fig 2 shows the Relativistic effect on the Doppler signal for the same mission.

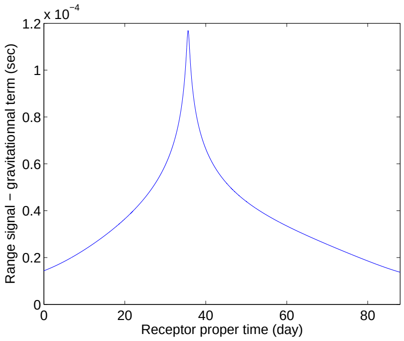

Figure 1: Relativistic effect on the Range signal for the BepiColombo mission.

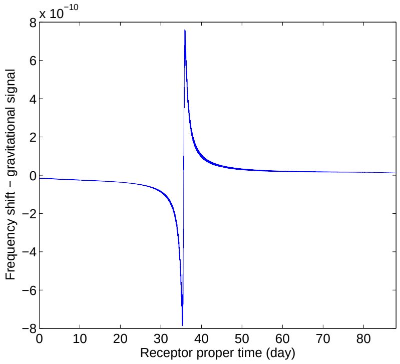

Figure 2: Relativistic effect on the Doppler signal for the BepiColombo mission.

## 4. Summary and Conclusions

We present a software that generically determine the Range/Doppler signal directly from the metric. This approach is slower than the usual approach but is interesting to have an idea of the order of magnitude of such effects. This approach is also very useful if we want to change the gravity theory since the only thing to change is the metric. With this software, it is very easy to see the impact of the gravity theory on the signal, and the difference between alternative theories.

A complete understanding of the physically observable effects also requires the adjustment of parameters in the standard theory (GR) to the observables simulated in a different theory using our software. This will allow detection of effects on the residuals and the determination of correlations between fundamental effects and other adjusted parameters. Such a study is the aim of our project over the next years.

## Acknowledgements

A. Hees is research fellow from FRS-FNRS (Belgian Fund for Scientific Research) and he thanks FRS-FNRS for financial support for his thesis at ORB-UCL (Observatoire Royal de Belgique - Université Catholique de Louvain, Belgium). Numerical simulations were made on the local computing ressources (Cluster URBM-SYSDYN) at the University of Namur (FUNDP, Belgium).

## References

[1] Misner, C. W., Thorne, K. S., and Wheeler, J. A: Gravitation, San Francisco: W.H. Freeman and Co., 1973

[2] Le Poncin-Lafitte, C., Linet, B., Teyssandier, P.: World function and time transfer: general post-Minkowskian expansions, Classical and Quantum Gravity, 21, 4463, sep, 2004

[3] Teyssandier, P., Le Poncin-Laffite, C.: General post-Minkowskian expansion of time transfer functions, Classical and Quantum Gravity, 25, 145020, jul 2008

[4] Balog, A. et al.:BepiColombo: an interdisciplinary Cornerstone Mission to the Planet Mercury, ESA-SCI(2000)1,2000

[5] Milani, A., Vokrouhlický, D., Villani, D., Bonanno, C., Rossi, A., Testing general relativity with the BepiColombo radio science experiment, Phys. Rev. D, 66, 082001, oct 2002