

3D Radiative Transfer in Titan's Atmosphere

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Titan presents some unique challenges to the radiative transfer modeler. Its atmosphere is the most extended in the solar system and the usual plane parallel assumption cannot be employed in most cases. Moreover, the scattering of solar radiation by aerosols and fluorescent scattering in molecular band occur in the same levels on Titan. We describe below an approach to solution of the radiative transfer equation that can deal with both these challenges.

The radiative transfer in spherical geometry can be written as

$$\frac{dI(\vec{r}, \hat{\Omega}, \nu)}{ds} = -k(\nu) (I(\vec{r}, \hat{\Omega}, \nu) - S(\vec{r}, \hat{\Omega}, \nu)) \quad (1)$$

where ν is the frequency, I is the specific intensity, S is the source function, k is the extinction coefficient and s is the differential path length along direction $\hat{\Omega}$. For the situation where there is both monochromatic scattering by dust and scattering a molecular band, which redistributes frequencies, the source function is given by

$$\begin{aligned} S(\vec{r}, \hat{\Omega}, \nu) &= \frac{w^d k^d(\nu)}{4\pi k(\nu)} \int p(\hat{\Omega} \cdot \hat{\Omega}') I(\vec{r}, \hat{\Omega}', \nu) d\hat{\Omega}' \\ &+ \frac{w^d k^d(\nu)}{4\pi k(\nu)} p(\hat{\Omega} \cdot \hat{\Omega}_o) F(\nu) \exp(-\tau(\vec{r}, \hat{\Omega}_o, \nu)) \\ &+ \frac{w^g k^g(\nu)}{4\pi k(\nu)} \frac{B(\nu)}{B_o} \int \int \frac{k^g(\nu')}{k_o^g} p(\hat{\Omega} \cdot \hat{\Omega}') I(\vec{r}, \hat{\Omega}', \nu') d\hat{\Omega}' d\nu' \\ &+ \frac{w^g k^g(\nu)}{4\pi k(\nu)} \frac{B(\nu)}{B_o} p(\hat{\Omega} \cdot \hat{\Omega}_o) \int \frac{k^g(\nu')}{k_o^g} F(\nu') \exp(-\tau(\vec{r}, \hat{\Omega}_o, \nu')) d\nu' \end{aligned} \quad (2)$$

where $k^g(\nu)$ is the extinction coefficient for absorption in the molecular band, $k^d(\nu)$ is the extinction coefficient for dust, and $k(\nu)$ is the sum. I is the intensity, B is the planck function, F is the solar flux, $-\Omega_o$ is the direction of the Sun, p is the phase function for aerosol scattering, and ω^g and ω^d are the single scattering albedos for gas and dust. The quantity k_o^g is the band-integrated molecular absorption coefficient, defined by

$$k_o^g = \int_{band} k^g(\nu') d\nu' \quad (3)$$

and B_o is the mean Planck function for the band, given by

$$B_o = \frac{1}{k_o^g} \int_{band} d\nu' k^g(\nu') B(\nu'). \quad (4)$$

The formal solution to equation 4 is

$$I(\vec{r}, \hat{\Omega}, \nu) = \int_0^\infty S(\vec{r}(s), \hat{\Omega}, \nu) \exp(-\tau(s, \hat{\Omega}, \nu)) k(\vec{r}(s), \nu) ds + I(\vec{r}_b, \hat{\Omega}, \nu) \exp(-\tau(\vec{r}_b, \hat{\Omega}, \nu)) \quad (5)$$

where \vec{r}_b is the location of the boundary in the $-\hat{\Omega}$ direction. We solve the coupled equations 5 and 8 by iteration starting with

$$\begin{aligned} S^0(\vec{r}, \hat{\Omega}, \nu) &= \frac{w^d k^d(\nu)}{4\pi k(\nu)} p(\hat{\Omega} \cdot \hat{\Omega}_o) F(\nu) \exp(-\tau(\vec{r}, \hat{\Omega}_o, \nu)) \\ &+ \frac{w^g k^g(\nu)}{4\pi k(\nu)} \frac{B(\nu)}{B_o} p(\hat{\Omega} \cdot \hat{\Omega}_o) \int \frac{k^g(\nu')}{k_o^g} F(\nu') \exp(-\tau(\vec{r}, \hat{\Omega}_o, \nu')) d\nu' \end{aligned} \quad (6)$$

followed by

$$I^{m+1}(\vec{r}, \hat{\Omega}, \nu) = \int_0^\infty S^m(\vec{r}(s), \hat{\Omega}, \nu) \exp(-\tau(s, \hat{\Omega}, \nu)) k(\vec{r}(s), \nu) ds + I(\vec{r}_b, \hat{\Omega}, \nu) \exp(-\tau(\vec{r}_b, \hat{\Omega}, \nu)) \quad (7)$$

and

$$\begin{aligned} S^m(\vec{r}, \hat{\Omega}, \nu) &= \frac{w^d k^d(\nu)}{4\pi k(\nu)} \int p(\hat{\Omega} \cdot \hat{\Omega}') I^m(\vec{r}, \hat{\Omega}', \nu) d\hat{\Omega}' \\ &+ \frac{w^d k^d(\nu)}{4\pi k(\nu)} p(\hat{\Omega} \cdot \hat{\Omega}_o) F(\nu) \exp(-\tau(\vec{r}, \hat{\Omega}_o, \nu)) \\ &+ \frac{w^g k^g(\nu)}{4\pi k(\nu)} \frac{B(\nu)}{B_o} \int \int \frac{k^g(\nu')}{k_o^g} p(\hat{\Omega} \cdot \hat{\Omega}') I^m(\vec{r}, \hat{\Omega}', \nu') d\hat{\Omega}' d\nu' \\ &+ \frac{w^g k^g(\nu)}{4\pi k(\nu)} \frac{B(\nu)}{B_o} p(\hat{\Omega} \cdot \hat{\Omega}_o) \int \frac{k^g(\nu')}{k_o^g} F(\nu') \exp(-\tau(\vec{r}, \hat{\Omega}_o, \nu')) d\nu' \end{aligned} \quad (8)$$

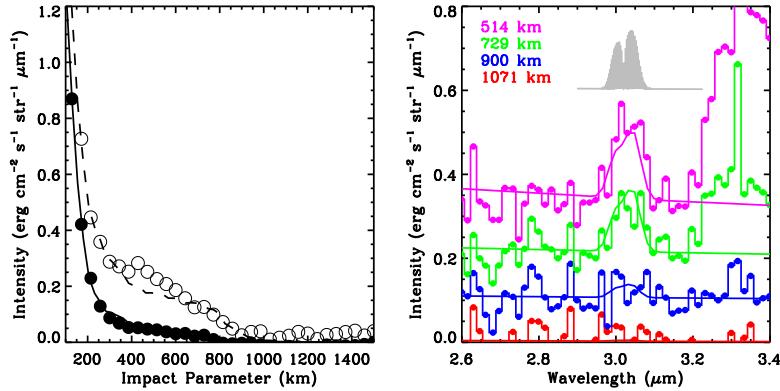


Figure 1: Preliminary results from the 3D radiative transfer model. The lefthand panel shows the integrated intensity in the HCN band at $3.0 \mu\text{m}$ (open symbols and dashed line) and in the continuum at $3.1 \mu\text{m}$ (closed symbols and solid line) as a function of the impact parameter of the line of sight. The righthand panel shows the calculated and observed spectra at several altitudes.