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Some particularities of the Titan translatory - rotary motion caused by third harmonic of its gravitational potential

Yu.V. Barkin
Sternberg Astronomical Institute, Moscow, Russia/barkin@inbox.ru/Fax:+07-095-9328841

Abstract. The third harmonic of force function of the synchronous satellite (Titan) leads to constant angular displacements of its main central axes of inertia w.r.t directions radius-vector, normals and a tangent to an orbit. In work it is shown, that in case of the elementary circular orbit of the synchronous satellite (Titan) the specified angular displacement of a pole of the greatest axis of inertia ellipsoid of inertia on a longitude σ (in the plane of circular orbit) and in latitude φ (relatively to orbital plane) consist $\sigma = 30''69$ и $\varphi = 5''54$, accordingly.

1 Introduction. In the paper [1] on the base of precise radio tracking of the spacecraft Cassini has provided a determination of Titan's mass and gravity harmonics to degree 3.

Table 1. The 3 × 3 gravity field of Titan, estimated from combined solutions using different approaches (for unnormalized spherical harmonics, reference radius 2575 km).

	Multi-arc (SOL1) [value ± 1σ (×10^{+06})]	Global (SOL2) [value ± 1σ (×10^{+06})]
J_2	31.808 ± 0.404	33.462 ± 0.632
C_{21}	0.338 ± 0.350	0.048 ± 0.115
S_{21}	−0.352 ± 0.438	−0.620 ± 0.496
C_{22}	9.983 ± 0.039	10.022 ± 0.071
S_{22}	0.217 ± 0.041	0.256 ± 0.072
J_3	−1.879 ± 1.019	−0.074 ± 1.051
C_{31}	1.058 ± 0.260	1.805 ± 0.297
S_{31}	0.509 ± 0.202	0.283 ± 0.354
C_{32}	0.364 ± 0.113	0.136 ± 0.158
S_{32}	0.347 ± 0.080	0.159 ± 0.105
C_{33}	−0.199 ± 0.009	−0.185 ± 0.012
S_{33}	−0.171 ± 0.015	−0.149 ± 0.016

It was shown that the quadrupole field is consistent with a hydrostatically relaxed body shaped by tidal and rotational effects. The inferred moment of inertia factor is about 0.34, implying incomplete differentiation, either in the sense of imperfect separation of rock from ice or a core in which a large amount of water remains chemically bound in silicates. Parameters of second and third harmonics of gravitational field are presented in Table 1.

In the work for studying some fine effects in rotary motion of the Titan parameters of first of the models presented by the table (SOL1) are used.

2 Orientation of the principal equatorial axes of inertia. Let $Oxyz$ and $Ox_p y_p z_p$ are two central Cartesian systems of coordinates of Titan. Axes of system of coordinates $Ox_p y_p z_p$ are orientated on the principal axes of inertia of a planet. We shall concentrate attention to displacement of poles of principal equatorial axes of inertia, believing, that axes Oz and Oz_p coincide with a polar axis of inertia of Tutan. The principal equatorial axis Ox_p appropriate to the least moment of inertia of a planet A_p, is revolved in a coordinate plane Oxy on some angle λ_0. Other two principal axes Oy_p and Oz_p also correspond to a middle B_p and greatest C_p of the principal moments of inertia ($C_p > B_p > A_p$). We shall designate through C_{nk}, S_{nk} and C_{nk}^p, S_{nk}^p Stoks constants of the planet gravitational field calculated in the basic planetary system of coordinates $Oxyz$ and in the principal axes of inertia $Ox_p y_p z_p$. Errors in determination of coefficients C_{21} and S_{21} are enough greater (Table 1), therefore in the given work as them пренебрежем. Thus values of coefficients we shall accept $C_{21} = S_{21} = 0$.

At the made assumptions of a choice of systems of coordinates between these constants are obtained simple relations:

$$C_{nk}^p = C_{nk} \cos(k\lambda_0) + S_{nk} \sin(k\lambda_0),$$

$$S_{nk}^{p} = -C_{nk}\sin(k\lambda_0) + S_{nk}\cos(k\lambda_0). \quad (1)$$

Value of an angle λ_0 can be determined unequivocally on values of two coefficients of a geopotential C_{22} and S_{22} from conditions $S_{22}^{p} = 0$ and $C_{22}^{p} > 0$. Using equality (2), we obtain: $\tan(2\lambda_0) = S_{22}/C_{22}$ and formula for error:

$$\Delta\lambda_0 = \frac{C_{22}\Delta S_{22} + S_{22}\Delta C_{22}}{2C_{22}^2}\cos^2(2\lambda_0).$$

For principal axes of inertia we have (1 unit=10^{-6}):

$$C_{22}^{p} = C_{22}\cos(2\lambda_0) + S_{22}\sin(2\lambda_0) = 9.983.$$

We shall accept for coefficients of potential of Titan C_{22} and S_{22} values: $C_{22} = (9.983 \pm 0.039)\cdot 10^{-6}$ and $S_{22} = (0.217 \pm 0.041)\cdot 10^{-6}$ ([1], model SOL1).

In result it is obtained, that the pole of the principal equatorial axis of inertia Ox_p of Titan has a longitude $\lambda_0 = (0^0 62 \pm 0^0 12)E$. Accordingly the pole of other equatorial axis corresponds to middle principal moment of inertia and it is position is characterized by a longitude $\lambda_0 = (90^0 62 \pm 0^0 12)E$.

Now values of coefficients of the third harmonic can be calculated directly in the main axes of inertia $Ox_p y_p z_p$ on formulae (1) (1 unit=10^{-6}):

$$\begin{aligned}
C_{31}^{p} &= C_{31}\cos(\lambda_0) + S_{31}\sin(\lambda_0) = 1.064,\\
S_{31}^{p} &= -C_{31}\sin(\lambda_0) + S_{31}\cos(\lambda_0) = 0.498,\\
C_{32}^{p} &= C_{32}\cos(2\lambda_0) + S_{32}\sin(2\lambda_0) = 0.371,\\
S_{32}^{p} &= -C_{32}\sin(2\lambda_0) + S_{32}\cos(2\lambda_0) = 0.339,\\
C_{33}^{p} &= C_{33}\cos(3\lambda_0) + S_{33}\sin(3\lambda_0) = -0.205,\\
S_{33}^{p} &= -C_{33}\sin(3\lambda_0) + S_{33}\cos(3\lambda_0) = -0.171.
\end{aligned} \quad (2)$$

3 Fine effects in Titan orientation. As it has been shown by the author the third harmonic of force function of the synchronous satellite leads to constant angular displacements of its principal central axes of inertia relatively to directions of the radius-vector, normal and a tangent to an orbit. So in case of the elementary circular orbit of the synchronous satellite (Titan) the specified angular displacement of a pole of the greatest axis of inertia of ellipsoid of inertia on longitude σ (in the plane of circular orbit) and in latitude φ (relatively to orbital plane) are discribed by formulae [2]:

$$\sigma = \frac{S_{31} - 30S_{33}}{8C_{22}}\cdot\frac{r}{a}, \quad \varphi = \frac{3(10C_{32} - C_{30})}{8(2C_{22} - C_{20})}\cdot\frac{r}{a}. \quad (3)$$

Here coefficients of the second and third harmonics correspond to the principal axes of inertia of the satellite. $r = 2575$ km is the mean radius of Titan, $a = 1221900$ km is the radius of orbit (in its simplified model presentation). For values of parameters (2) on formulas (3) we find the values of angles:

$$\sigma = 30''63, \quad \varphi = 5''77.$$

Along with significant effects in rotary motion of the Titan, described above there is one more exclusively fine (small) effect according to which because of influence of the third harmonic of force function the stationary circular orbit of the satellite "rises" above a plane of not indignant circular orbit on distance

$$z = -\frac{3}{8}(10C_{32} - C_{30})\cdot\left(\frac{r}{a}\right)^3 a = 1.2\cdot 10^{-3}\ \text{mm}.$$

In other words the center of a circular orbit of the Titan (in the modelling solution) is not center of mass of central planet (Saturn), and point some close but displaced in space. Certainly, in the given work we neglect eccentricity of orbit of the Titan and other orbital perturbations.

In summary we shall note, that earlier similar effects of non-classical stationary translatory-rotary motions of the satellite have been revealed in motion of the Moon and Phobos [2]. In case of the Moon, the angular displacements of the greater of axes of inertia make: $\sigma = 274''$ and $\varphi = 75''$, that proves to be true data of observations, and estimation of small translational displacement of the Moon makes $z = 0.8$ mm . For Phobos mentioned effects are evaluated by following values: $\sigma = 20''0$ and $\varphi = -14''1$ and $z = 2.7$ mm [2]. The description and history of a discovery of similar solutions are resulted in work [3]. Below the endurance from mentioned paper is given (p. 3).

[3] O'Reilly O.M., Thoma B.L. (2003).

> In recent works on rigid satellites in gravitational fields, motions of the center of mass whose orbital plane does not contain the center of mass of the secondary have been discovered. The motions are in contrast to the classical motions studied by Lagrange and many others where the centers of mass of the satellite and primary both lie on the orbital plane.[4] It is usual to consider a standard approximation to the gravitational potential in models for the dynamics of the satellite-secondary system. However, Abul'naga and Barkin (1979) discovered the non-Lagrangian orbits by considering higher-order terms in the asymptotic expansion for the gravitational potential U of the rigid satellite. On the other hand, the independent discovery of these orbits by Wang et al. (1991, 1992), Abrarova (1995) and Abrarova and Karapetyan (1996) was facilitated by their consideration of satellites composed of rigidly connected discrete mass particles; thereby enabling use of the exact expression for U.
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> [4] The non-classical motions are called non-Lagrangian by Barkin (1985), non-great-circle orbits by Wang et al. (1991, 1992), and non-trivial by Abrarova (1995) and Abrarova and Karapetyan (1996).

In the future as a result of determination of coefficients of gravitational potentials by the third and higher harmonics the phenomena of constant angular displacements of axes of inertia considered here and "phenomenon of splitting" of a plane of unperturbed circular orbit will be revealed and described in motion of all synchronous satellites in Solar and other planetary systems.

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References

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[**2**] Barkin, Yu.V. (1980) Peculiarities in the moon's translational-rotational motion due to the third and higher harmonics in its force function. (Pis'ma v Astronomicheskii Zhurnal, vol. 6, June 1980, pp. 377-380.) Soviet Astronomy Letters, vol. 6, May-June 1980, pp. 208-210. Translation. In Russian. In Russian.

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