EPSC Abstracts
Vol. 5, EPSC2010-734, 2010
European Planetary Science Congress 2010

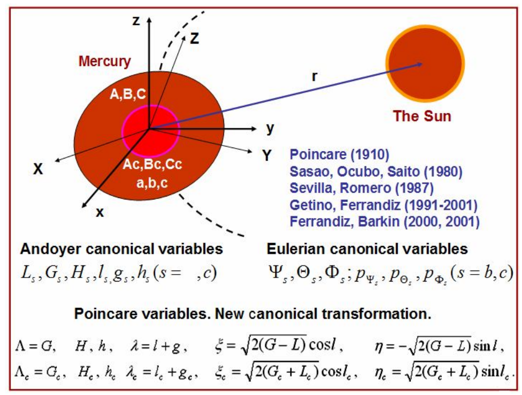
To development of accurate theory of Mercury rotation by accurate description of its orbital motion

Yu.V. Barkin (1), J.M. Ferrandiz (2)
(1) Sternberg Astronomical Institute, Moscow, Russia (barkin@inbox.ru / Fax: +07-495-9328841)
(2) Alicante University, department of applied mathematics

Abstract. In the work the analytical theory of forced librations of the Mercury considered as a rigid non-spherical celestial body (in general case with liquid core) is developed. By construction of developments of the force function of the problem we take into account nesessary planetary perturbations in orbital motion of planet. For the basic variables: Andoyer, Poincare and Eulerian angles (Slide 1), and also for various dynamic characteristics of Mercury the tables for amplitudes, periods and phases of perturbations of the first order have been constructed.

Harmonic development of disturbing functions in theories of rotation of the solar system bodies. In known theories of the Earth rotation (Kinoshita, Souchay, Ferrandiz, Getino and other colleagues, 1977 - 2002) by construction of developments of the second harmonic of force function in trigonometric series in Andoyer variables have been used some restricting assumptions about properties of orbital perturbations of the Moon and the Sun motions. Similar semplifications have been used by construction of analytical theories of the Moon and Mercury rotation in our earlier papers (Barkin, 1986; Barkin, Ferrandiz, 2004). For more accurate description of the treatment of the problems about rotational motion of the Solar system bodies we have developed new approach to construction of the developments of the force function in form of Poinsot series. And by that we remove restrictions and discrepancies in statements of problems about rotary motions of celestial bodies entered into mentioned works [1]. In the paper for description of rotation of Mercury we use Andoyer variables: ρ angle of inclination of angular momentum vector of Mercury $\mathbf{G}$ with respect to normal to base plane of reference system of date and h is a longitude of node of intermediate Andoyer plane orthogonal to vector $\mathbf{G}$ (Slide 1). θ, l and g are Eulerian angles, determining orientation of principal axis of inertia of Mercury with respect to intermediate reference system connected with angular momentum vector.

Slide 1.



Base parameters of the problem (of the force function) are: coefficients of second harmonic of Mercury gravitational potential J_2 and C_{22}, dimensionless moment of inertia of Mercury I and dynamical oblateness $\delta = 4C_{22}/(J_2 + 2C_{22})$. In result on the base developed approach [1] we have obtain following trigonometric development of the force function of gravitational interaction non-spherical Mercury and the Sun:

$$
\begin{aligned}
U = -\frac{3}{2} n^2 m R^2 \left(J_2 + 2C_{22}\right) \Big\{ & \mathrm{N}_0 \sum_{\mathbf{v}} \left[R_{0,\mathbf{v}}(\rho,t)\cos\Theta_{\mathbf{v}} + r_{0,\mathbf{v}}(\rho,t)\sin\Theta_{\mathbf{v}} \right] + \\
& + \sum_{\mu} N_{0,2\mu} \sum_{\mathbf{v}} \left[R_{0,\mathbf{v}}(\rho,t)\cos\left(\Theta_{\mathbf{v}} - 2\mu l\right) - r_{0,\mathbf{v}}(\rho,t)\sin\left(\Theta_{\mathbf{v}} - 2\mu l\right) \right] + \\
& + \mathrm{N}_{1,0} \sum_{\varepsilon} \sum_{\mathbf{v}} \left[R_{1,\mathbf{v}}^{(\varepsilon)}(\rho,t)\cos\left(g - \varepsilon\Theta_{\mathbf{v}}\right) + r_{1,\mathbf{v}}^{(\varepsilon)}(\rho,t)\sin\left(g - \varepsilon\Theta_{\mathbf{v}}\right) \right] + \qquad (1) \\
& + \mathrm{N}_{2,0} \sum_{\varepsilon} \sum_{\mathbf{v}} \left[R_{2,\mathbf{v}}^{(\varepsilon)}(\rho,t)\cos\left(2g - \varepsilon\Theta_{\mathbf{v}}\right) + r_{2,\mathbf{v}}^{(\varepsilon)}(\rho,t)\sin\left(2g - \varepsilon\Theta_{\mathbf{v}}\right) \right] + \\
& + \sum_{\mu} \mathrm{N}_{1,\mu} \sum_{\varepsilon} \sum_{\mathbf{v}} \left[R_{1,\mathbf{v}}^{(\varepsilon)}(\rho,t)\cos\left(g - \varepsilon\Theta_{\mathbf{v}} + 2\mu l\right) + r_{1,\mathbf{v}}^{(\varepsilon)}(\rho,t)\sin\left(g - \varepsilon\Theta_{\mathbf{v}} + 2\mu l\right) \right] + \\
& + \sum_{\mu} \mathrm{N}_{2,\mu} \sum_{\varepsilon} \sum_{\mathbf{v}} \left[R_{2,\mathbf{v}}^{(\varepsilon)}(\rho,t)\cos\left(2g - \varepsilon\Theta_{\mathbf{v}} + 2\mu l\right) + r_{2,\mathbf{v}}^{(\varepsilon)}(\rho,t)\sin\left(2g - \varepsilon\Theta_{\mathbf{v}} + 2\mu l\right) \right] \Big\}.
\end{aligned}
$$

$$
\varepsilon = \pm 1, \ \mu = \pm 1; \ \Theta_{\mathbf{v}} = \nu_1 L_{Me} + \nu_2 L_V + \nu_3 L_E + \nu_4 L_{Ma} + \nu_5 L_{Ju} + \nu_6 L_{Sa} + \nu_7 L_{Ur} + \nu_8 L_{Ne},
$$

$$\mathrm{N}_0=(2+\delta)\left(2-3\sin^2\theta\right)/2\,,\mathrm{N}_{0,2\mu}=-3\delta\sin^2\theta/4\,,\mathrm{N}_{1,0}=\sin 2\theta(2-\delta)/8\,,N_{2,0}=\sin^2\theta(\delta-2)/8\,.$$

Functions N_0, $\mathrm{N}_{0,2\mu}$, $\mathrm{N}_{1,0}$ and $\mathrm{N}_{2,0}$ depend only from Andoyer angle θ and dynamical parameter δ. Arguments in $\Theta_{\mathbf{v}}$ located on multiple of mean longitudes of planets (Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uran and the Neptune); $\mathbf{v}=(\nu_1,\nu_2,\nu_3,...,\nu_8)$ are corresponding sets of integer indexes. Here all functions R and r are special inclination functions depending from angle ρ of inclination of vector of angular momentum of Mercury with respect to normal to base (Laplace) plane and coefficients: $A_{\mathbf{v}}^{(j)}$, $B_{\mathbf{v}}^{(j)}$ and $a_{\mathbf{v}}^{(j)}$, $b_{\mathbf{v}}^{(j)}$:

$$R_{0,\mathbf{v}}(\rho,t)=-\frac{1}{6}\left(3\cos^2\rho-1\right)A_{\mathbf{v}}^{(0)}-\frac{1}{2}\sin 2\rho A_{\mathbf{v}}^{(1)}-\frac{1}{4}\sin^2\rho A_{\mathbf{v}}^{(2)}\,,$$

$$R_{1,\mathbf{v}}^{(\varepsilon)}=\sin 2\rho\left(A_{\mathbf{v}}^{(0)}-\frac{1}{2}A_{\mathbf{v}}^{(2)}\right)-2\cos 2\rho A_{\mathbf{v}}^{(1)}+2\varepsilon\cos\rho B_{\mathbf{v}}^{(1)}-\varepsilon\sin\rho B_{\mathbf{v}}^{(2)}\,,$$

....

$$r_{2,\mathbf{v}}^{(\varepsilon)}=2\sin\rho b_{\mathbf{v}}^{(1)}+\cos\rho b_{\mathbf{v}}^{(2)}-\varepsilon a_{\mathbf{v}}^{(2)}-\varepsilon\sin^2\rho\left(a_{\mathbf{v}}^{(0)}-\frac{1}{2}a_{\mathbf{v}}^{(2)}\right)+\varepsilon\sin 2\rho a_{\mathbf{v}}^{(1)}\,.$$

Coefficients $A_{\mathbf{v}}^{(j)}$, $B_{\mathbf{v}}^{(j)}$ and $a_{\mathbf{v}}^{(j)}$, $b_{\mathbf{v}}^{(j)}$ with high accuracy have been presented as quadratic functions of the time which take into account secular planetary perturbations in the Mercury orbital motion (Kudrjavsev, 2009; Barkin, Kudrjavsev, Barkin, 2009):

$$\mathrm{A}_{\mathbf{v}}^{(j)}=\mathrm{A}_{\mathbf{v};0}^{(j)}+\mathrm{A}_{\mathbf{v};1}^{(j)}\cdot t+\mathrm{A}_{\mathbf{v};2}^{(j)}\cdot t^2\,,\ \mathrm{A}=(A,B,a,b)\,,\ j=(0,1,2)\,.$$

These coefficients generalize similar Kinoshita's coefficients (in Earth rotation theory) and represent full and exact developments of following functions of heliocentric spherical coordinates of Mercury (r, φ and λ):

$$\frac{1}{2}\left(\frac{a}{r}\right)^3\left(1-3\sin^2\varphi\right)=\sum_{\mathbf{v}}A_{\mathbf{v}}^{(0)}\cos\Theta_{\mathbf{v}}+a_{\mathbf{v}}^{(0)}\sin\Theta_{\mathbf{v}}\,,$$

$$\left(\frac{a}{r}\right)^3\cos^2\varphi\cos 2\left(\lambda-h\right)=\sum_{\mathbf{v}}A_{\mathbf{v}}^{(2)}\cos\Theta_{\mathbf{v}}+a_{\mathbf{v}}^{(2)}\sin\Theta_{\mathbf{v}}\,,$$

$$\left(\frac{a}{r}\right)^3\cos^2\varphi\sin 2\left(\lambda-h\right)=\sum_{\mathbf{v}}B_{\mathbf{v}}^{(2)}\sin\Theta_{\mathbf{v}}+b_{\mathbf{v}}^{(2)}\cos\Theta_{\mathbf{v}}\,,$$

$$\left(\frac{a}{r}\right)^3\sin\varphi\cos\varphi\sin\left(\lambda-h\right)=\sum_{\mathbf{v}}A_{\mathbf{v}}^{(1)}\cos\Theta_{\mathbf{v}}+a_{\mathbf{v}}^{(1)}\sin\Theta_{\mathbf{v}}\,,$$

$$\left(\frac{a}{r}\right)^3\sin\varphi\cos\varphi\cos\left(\lambda-h\right)=\sum_{\mathbf{v}}B_{\mathbf{v}}^{(1)}\sin\Theta_{\mathbf{v}}+b_{\mathbf{v}}^{(1)}\cos\Theta_{\mathbf{v}}\cdot$$

The new expansions are valid over 2000 years, 1000AI 3000AD, have a form similar to that of Kinoshita's series. The latest long-term numerical ephemerides of the Moon and planets DE-406 are used as the source of disturbing bodies coordinates. The mentioned developments have been constructed not only for the problem about Mercury rotation but also for the problems about Earth rotation, Venus rotation and in theory of the Moon rotation (Barkin, Kudrjavsev, Barkin, 2009). Corresponding developments of Kinoshita in the Earth rotation theory are obtained as particular case from above mentioned formulae by restricting conditions: $r=a=b=0$.

Construction of theory of Mercury librations. On the base of obtained development of force function the perturbations of the first order in Mercury librations in longiude, in inclination (and for wide grope of another variables: Andoyer and Poincare variables, components of angular velocity and oth.) have been obtained and analized in the work. The Barkin's and Kudrjavsev's work partially was accepted by RFBR grant N 08-02-00367.

References

[1] Barkin Yu.V., Kudrjavsev C.M., Barkin M.Yu. (2009) Perturbations of the first order of the Moon rotation. Proceedings of International Conference "Astronomy and World Heritage: across Time and Continents" (Kazan, 19-24 August). KSU, Section C: "The Moon, moons and planets: Robotic Explorations and Comparison, pp. 161-164.