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Determination of Lunar physical libration from polar observations in the project ILOM (Japan)

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Abstract

In the frame of the second stage of the Japanese space mission SELENE-2 [1] the project ILOM (In-situ Lunar Orientation Measurement) planned after 2017 year is a kind of instrument for positioning on the Moon. It will be set near the lunar pole and will determine parameters of lunar physical libration by positioning of several tens of stars in the field of view regularly for longer than one year. We found the method which allows determining of libration angles in the node Iσ and in inclination ρ with certain accuracy. It's shown that libration in longitude τ cannot be determined from observation of polar stars, because polar distance of a star is not sensitive to longitudinal oscillation of a Lunar body.

1. Setting up a problem

The project ILOM proposes that polar distance will be measured for every star crossing lunar prime meridian. Stars were taken from various stellar catalogues, such as the UCAC2-BSS, Hipparcos, Tycho and FK6. Stellar coordinates α_o, δ_0 were reduced from ICRF system to the ecliptical one λ, β at the epoch of observation [2]. Principal axes of inertia were taken as a frame for selenography system coordinates (Fig. 1) [3]. Reduction of the ecliptical coordinates to the selenography one (δ, α) were done using the matrixes $\prod_{i}(a)$ of rotation on the angle *a* relatively axis *i* :

$$\begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} = \prod_{z}\left(\varphi + 180^{o}\right)\prod_{x}(-\Theta)\prod_{z}(\Psi)\begin{pmatrix} \cos\beta\cos\lambda \\ \cos\beta\sin\lambda \\ \sin\beta \end{pmatrix} \quad (1)$$

here (x, y, z) – axes of the main inertia, (φ, Θ, Ψ) – Eulerian angles, describing rotation of the Moon relatively ecliptical frame. They are determined by the libration angles: $\Theta(t) = I + \rho(t)$, $\varphi(t) = F + \tau(t)$, $\Psi(t) = \Omega + \sigma(t)$, where $\rho(t)$, $\tau(t)$, $\sigma(t)$ - are libration angles in the inclination, longitude and node. From observation we can measure polar distance of a star $p = \pi/2 - \delta$. We consider an ideal model, where the star is observed exactly in the prime meridian. In this case the selenography latitude $\alpha = 0$, and, as a consequence, $\cos\delta = p$.
As a result, in the system (1) the vectors (p, 0, $\sqrt{1-p^2}$)^T – is a vector of observations. Using it we need to find unknown values of libration angles at the moment of observation.

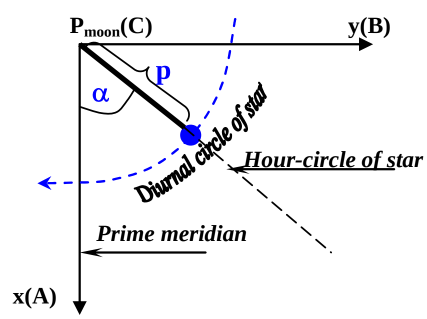

Figure 1: Selenography coordinates of a star near the lunar pole.

As a result, in the system (1) the vector (p, 0, $\sqrt{1-p^2}$)^T – is a vector of observations. Using it we need to find unknown values of libration angles $\rho(t), \sigma(t), \tau(t)$ at the moment of observation *t*. Let reduce the system (1) to the general form: $\boldsymbol{f}(X) = 0$
$\boldsymbol{f} = (f_1, f_2, f_3)^T$
where $X = \left(\rho(t), \sigma(t), \tau(t)\right)^T$ or $X = \left(x_1, x_2, x_3\right)^T$ (2)

The Jacobian of the system (2)

$$J(X) = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f\,2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{vmatrix} \approx 0 \quad (3)$$

This means, that the system (2) is ill-posed and, as a result, the well-known Newton method for solution of non-linear algebraic equations system does not give convergent solution [4].

2. Gradient method for the solution of libration equations

For described case we have found one of optimization methods – gradient method [5]. It's based on the minimization of the function $\Phi(X)=\sum_{i=1}^{3} f_i^2(X)$. As a result, we do not need to use the inverse Jacobian (3) and the iteration procedure for the system (2) is well-convergent.

To start the iteration procedure we use as initial solution the data, calculated by analytical theory of physical libration [4], [6]. We simulated observation error and followed its influence on the libration angles. At the Fig. 2 we can see that the errors in polar distance $|\Delta p| \le 0.010''$ causes the same error in $\Delta\rho$ and $\Delta I\sigma$. At the same time value of τ is independent on polar distance and cannot be determined from polar stars.

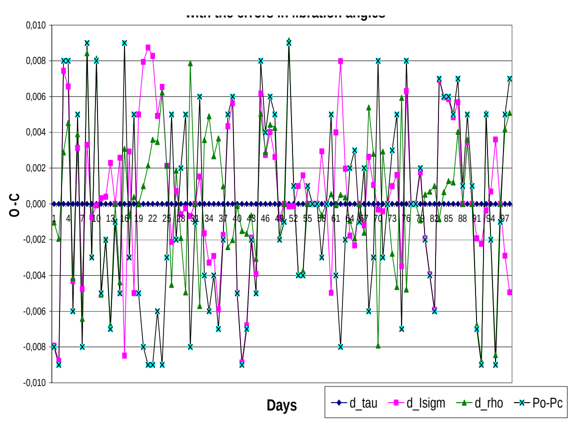

Figure 2: Correlation of errors in observed polar distance (Po) with the errors in libration angles

Investigating of the total increment and differential of the functions (2) allowed us to write the following relations between errors in libration angles and in polar distance (p): $DD(t,\tau_c,\rho_c,\sigma_c)\times\Delta\tau=\Delta p$; $Dy(t,\tau_c,\rho_c,\sigma_c)\times\Delta\rho=\Delta p$; $Dz(t,\tau_c,\rho_c,\sigma_c)\times\Delta I\sigma=\Delta p$.

Numerical estimation of coefficients near libration angles gave us: $DD\approx 10^{-12}$; $Dy\approx 1$; $Dz\approx 1$. This confirms that libration in longitude does not correlate with polar distance. In fact, we can understand this phenomenon from geometry of physical libration: longitudinal librations depend on latitude δ proportionally to $\cos\delta$, which for the polar zone is close to zero.

3. Summary and Conclusions

We have found a suitable instrument for solution of "inverse problem" in the physical libration of the Moon: if we can measure the polar distance of a star with a high accuracy, then we can obtain libration angles in ρ and in Iσ at the moment of observation with the same accuracy. To improve accuracy of observations we suggest to measure a polar distance of a star several time before and after crossing the prime meridian. Our simulating is done only for the case when error in time of meridian transition is equal to zero. The next stage of our simulating should study this kind of errors in observation.

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