

Saturn's rings size distribution: Stochastic description of fragmentation of ring-agglomerates

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Abstract

We investigate the fragmentation dynamics of a two-dimensional agglomerate, held together by adhesive bonds, caused by an impacting projectile of given mass and impact speed/energy. The agglomerate is made of identical adhering spheres (constituents) forming a regular cubic lattice. A rather simple "random walk model" of a crack propagation has been studied numerically, where subsequent breaking of adhesive bonds (defining the crack path) is organized randomly until the impact energy of the projectile is exhausted. A large number of repeated numerical breakage experiments have yielded a fair agreement with egg-shell crushing lab experiments (Hermann et al., Physica A 371 (2006), 59) confirming the size distribution of the fragments obeying a power law, $p(s) \propto s^a$ with $a = -3/2$. We have been able to theoretically explain this size distribution by a one-dimensional random walk model mimicking the crack propagation. An analytical expression can be derived by applying the distribution of the first passages times characterizing the moments when two cracks meet (annihilate). An extension of this 2D model to three-dimensional aggregates is suggested by a two-dimensional (direction vector) random walk of the cracks. With these stochastic models we plan to describe the steady size distribution of the main population of Saturn's rings (centimeters up to a few meters in size) by a balance between coagulation and fragmentation.