

## Identification of asteroids in two-body and three-body mean motion resonances

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### Introduction

The resonance represents a commensurability between the mean frequencies of the orbital motions of an asteroid and two planets (e.g., Jupiter and Saturn):

$$m_J \dot{\lambda}_J + m_S \dot{\lambda}_S + m \dot{\lambda} \approx 0, \quad (1)$$

where  $\dot{\lambda}_J$ ,  $\dot{\lambda}_S$ ,  $\dot{\lambda}$  are the time derivatives of the mean longitudes of Jupiter, Saturn, and asteroid, respectively, and  $m_J$ ,  $m_S$ ,  $m$  are integers.

In order to distinguish between resonant and non-resonant motions, the *resonant argument* is introduced:

$$\sigma_{p_J, p_S, p} = m_J \lambda_J + m_S \lambda_S + m \lambda + p_J \varpi_J + p_S \varpi_S + p \varpi, \quad (2)$$

where  $\lambda_J$ ,  $\lambda_S$ ,  $\lambda$ ,  $\varpi_J$ ,  $\varpi_S$ ,  $\varpi$  are the mean longitudes and longitudes of perihelia of Jupiter, Saturn, and an asteroid, respectively, and  $m_J$ ,  $m_S$ ,  $m$ ,  $p_J$ ,  $p_S$ ,  $p$  are integers satisfying the d'Alambert rule:

$$m_J + m_S + m + p_J + p_S + p = 0. \quad (3)$$

- If resonant argument (2) librates, the system is in resonance.
- If it rotates, the system is out of resonance.

### Identification of resonances

As a first stage of the identification process we build an “identification matrix”. It consists of two basic columns. The first one contains designations of resonances (in the notation  $m_J m_S m(q)$ ), and the second one contains the resonant values of the semimajor axis.

At a second stage each asteroid’s orbit from the adopted set of 249567 objects (from AstDyS catalog) is computed for  $10^5$  years (taking into account the perturbations of all planets and Pluto). Each argument  $\sigma_{\text{res}}$  is then analyzed on the subject of libration/circulation. We distinguish two types of resonant

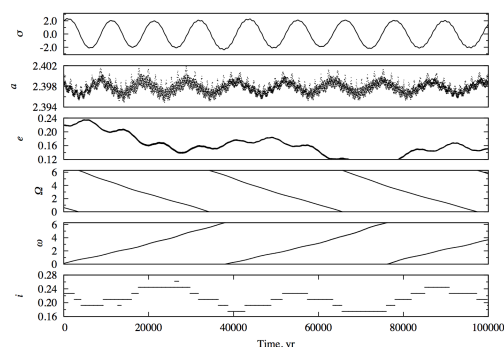


Figure 1: The resonant argument and orbital elements for 463 Lola, resonance 4 – 2 – 1.

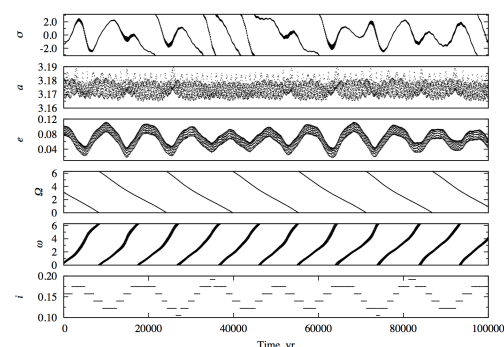


Figure 2: The resonant argument and orbital elements for 490 Veritas, resonance 5 – 2 – 2.

libration: pure and transient. The libration is pure, if it lasts during the whole time interval of integration (see Fig. 1). The libration is transient, if circulation appears at any time during this interval (see Fig. 2).

Table 1: An extract from the identification matrix for three-body resonances

$m_J$	$m_S$	$m$	$q$	$a_{\text{res}}$ (AU)
4	-2	-1	1	2.3981
5	-2	-2	1	3.1746
3	-2	-1	0	3.0801
3	-1	-1	1	2.7530
2	2	-1	3	2.6151

## Results of identification

Table 2: Asteroids in three-body resonances, statistics

	NM98 [5]	This study $q_{\text{max}} = 6$
Studied set of objects	5400	249567
Objects with integrated orbits	836	249567
Pure-resonant objects	—	2516
Transient+pure-resonant objects	255 (4.6%)	11295 (4.5%)
Objects in two-body pure resonances *	—	3213
Objects in two-body transient+pure resonances *	—	4537

\* Including Trojans and Hildas.

## Conclusions

- The fraction of asteroids in three-body resonances (transient plus pure) turns out to be  $\approx 4.5\%$  of the total studied set of 249567 asteroids. The fraction of asteroids in pure three-body resonances turns out to be  $\approx 1.0\%$  of the total studied set.
- The top three most populated three-body resonances are: 4 -2 -1 (692 resonant asteroids), 5 - 2 -2 (674), 3 -2 -1 (620).
- Except resonances 1/1 (Trojans) and 3/2 (Hildas), the top three most populated two-body resonances are: 11/5 (389 resonant asteroids), 2/1 (274), 8/3 (233).

## References

- [1] AstDyS website:  
<http://hamilton.dm.unipi.it/cgi-bin/astdys/>
- [2] Chambers J.: A hybrid symplectic integrator that permits close encounters between massive bodies, *Mon. Not. R. Astron. Soc.*, Vol. 304, pp. 793–799, 1999.
- [3] Morbidelli A.: *Modern Celestial Mechanics. Aspects of Solar System Dynamics*, Padstow, Taylor and Francis, 2002.
- [4] Murray N., Holman M. and Potter M.: On the origin of chaos in the asteroid belt, *Astron. J.*, Vol. 116, pp. 2583–2589, 1998.
- [5] Nesvorný D. and Morbidelli A.: Three-body mean motion resonances and the chaotic structure of the asteroid belt, *Astron. J.*, Vol. 116, pp. 3029–3037, 1998.
- [6] Nesvorný D. and Morbidelli A.: An analytic model of three-body mean motion resonances, *Celest. Mech. Dyn. Astron.*, Vol. 71, pp. 243–271, 1999.