

Long-period Librations of Mercury with an Inner Core

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Abstract

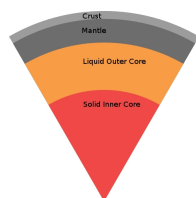
We compute the effect of the solid inner core on the long-period librations of the mantle of Mercury. These librations are due to the planetary perturbations on the orbit of Mercury. Depending of the internal structure, resonance can appear between one of the free modes and the planetary frequencies.

1. Introduction

The mantle of Mercury may experience different types of longitudinal librations: forced librations with annual (88d and its harmonics) periods, forced librations due to the planets and a free libration. The amplitude of the forced librations and the free libration frequency depend on the internal structure.

2. The interior model

We use updated versions of the interior models of Rivoldini et al (2009). These models have a crust, a mantle, a liquid outer core and a solid inner core. They are constrained by the mass and the radius of the planet.



Assumptions have to be made for the flattening of the internal layers.

3. The rotation model

Changes in angular momentum have to be considered for three layers of Mercury: the silicate shell (crust + mantle), the liquid outer core and the solid inner core. We take into account the torques on each part: the gravitational torque of the sun and gravitational and pressure torques between the layers.

$$\begin{aligned} \gamma_m'' - \sum_i \omega_i^2 \lambda_i \cos(\omega_i t + \phi_\lambda) &= -\omega_m^2 \gamma_m - \frac{2\Gamma}{C_m} (\gamma_m - \gamma_s) - \omega_m^2 \frac{f_1(e)}{2f_2(e)} \sin M \\ \gamma_s'' - \sum_i \omega_i^2 \lambda_i \cos(\omega_i t + \phi_\lambda) &= -\omega_s^2 \gamma_s + \frac{2\Gamma}{C_s} (\gamma_m - \gamma_s) - \omega_s^2 \frac{f_1(e)}{2f_2(e)} \sin M \end{aligned} \quad (1)$$

Planetary perturbations Coupling Annual forcing (2)

$\gamma_m(t)$ and $\gamma_s(t)$ are the libration angles of the mantle and of the solid inner core respectively, C_m and C_s their polar moments of inertia, M is the mean anomaly of Mercury, ω_i are the planetary frequencies, λ_i and ϕ_λ the amplitudes and phases of the planetary perturbations, Γ is the coupling parameter between the inner core and the silicate shell, $f_1(e)$ and $f_2(e)$ are functions of the eccentricity, ω_m and ω_s are the free modes of the two layers when there is no coupling.

3.1. Eigenfrequencies

There are 2 free modes (see Fig. 1). One of the mode has a period between 11.5y and 16y, the other is between 4y and 7y. These periods are in the range of

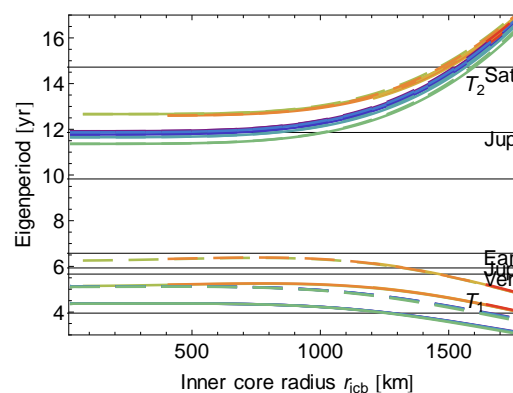


Figure 1: Periods of the 2 free modes as a function of the inner core size. The horizontal black lines show the main periods of the planetary perturbations. The different lines represent different interior model. The dashed lines assume an equatorial flattening of $\epsilon_f = \epsilon_m/2$ while the solid lines assume $\epsilon_f = \epsilon_m$.

periods of the planetary induced librations.

Here we assume a damped amplitude for the eigenmodes since the damping timescale is of the order of 10^5 years (Peale 2005).

Van Hoolst et al (2012) have computed the effect of the inner core on the amplitude of the annual forced libration (88d). It could be larger than the instrumental noise for inner core size larger than about 1000km.

4. Librations due to the planets

4.1. Without an inner core

If Mercury has only **2 layers** (a mantle and a liquid core) Dufey et al (2008), Peale et al (2009) and Yseboodt et al (2010) have shown that the amplitudes and phases of the long-period forced librations of Mercury's mantle can be computed from the planetary perturbations on the orbital element of Mercury (indirect effect).

Depending of the moment of inertia ratio value $(B - A)/C_m$, resonance can occur between the free mode and the 11.8y libration due to Jupiter.

4.2. With an inner core

For a **3 layer** planet (mantle, liquid outer core, solid inner core) the amplitude of the long-period librations is given by:

$$\gamma_{m_i} = \lambda_i \frac{\omega_g^2 + \omega_s^2 - \omega_i^2}{(\omega_1^2 - \omega_i^2)(\omega_2^2 - \omega_i^2)} \omega_i^2$$

ω_1 and ω_2 are the frequencies of the 2 free modes.

Again, resonances can occur, depending on the interior model (see Fig. 2).

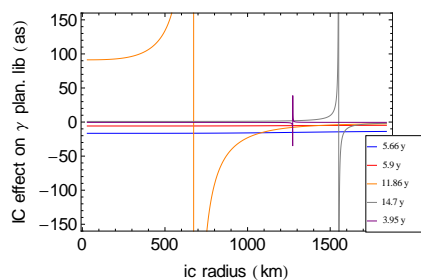


Figure 2: Amplitudes of the libration due to the planets, for one particular interior model.

5. Fit the radar data

Our rotation model for the mantle of Mercury now has the annual forced librations and also the long-period librations. Their amplitudes depend on the inner core size and flattening.

We use the measurement of the rotation rate of Mercury by Margot et al (2007) to search for the rotation model that best fits the radar data.

The results show a strong dependence on the $(B - A)/C_m$ ratio, while the presence of an inner core and the associated resonances can slightly change the best fit value.

6. Conclusion

- We give the expression for the amplitude of the long-period librations due to the planets when there is an inner core inside Mercury. Resonances can happen between the free modes and the long-period librations. The effect of the inner core on these long-period librations is not negligible, specially if the inner core is large.

- A large inner core considerably lengthens the free libration period.

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