

# Charged particles behaviour with real data on Mars

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## Abstract

Using classical source models it is possible to model the Martian crustal magnetization. In this context, the trajectories of different particles (electron and proton) in different magnetized sources have been analyzed by using a conservative numerical scheme. To get more realistic results gravitational effect and real values of the magnetic field obtained from MARSIS (MEX) data have been taken into account.

## 1. Introduction

The magnetic field of Mars is dominated by intensely magnetized sources distributed non-randomly in the Martian crust [1]. This remanent magnetic field was detected by the observations made on board Mars Global Surveyor. Connerney [2] suggested that the crustal magnetization could be explained by the use of source models, comparing the field produced by a volume of a magnetized material with the observed data. Thanks to the ionograms it is possible to obtain the values of the magnetic field at any specific height.

## 2. Equations

To obtain the trajectories of charged particles it has been used a numerical scheme with a conserved discrete kinetic energy and a time-inversion invariance. Starting from the equation of motion (1) and discretizing the system, it has been obtained a family of solutions from the follow scheme (2) in a matrix form.

$$m \frac{d\vec{v}}{dt} = \frac{e}{c} \vec{v} \times \vec{B} \quad (1)$$

$$\vec{v}^{n+1} - \vec{v}^n = \frac{e\hbar}{2mc} (\vec{v}^{n+1} + \vec{v}^n) \times \vec{B}^n \quad (2)$$

As a first approximation for the magnetic field we have used different kinds of homogenous sources like a sphere, a cylinder and a linear combination of both.

## 3. Numerical Results

Using the equations shown above and working with Gaussian units, it has been possible to get different trapped trajectories for an electron and a proton under different magnetic fields.

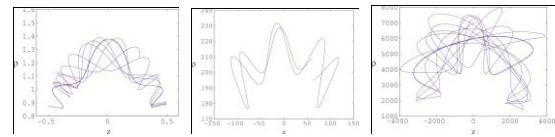


Figure 1: Electron trapped trajectories under different magnetic fields, sphere, cylinder, sphere+cylinder.

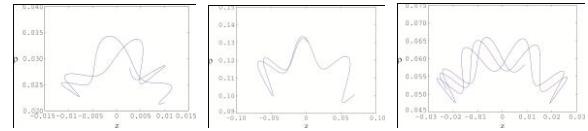


Figure 2: Proton trapped trajectories under different magnetic fields, sphere, cylinder, sphere+cylinder.

To get more realistic results, it has been included the effect of the gravity altitude-dependent gravity in the equation that is described by (3).

$$m \frac{d\vec{v}}{dt} = \frac{e}{c} \vec{v} \times \vec{B} - m \frac{GM}{r^2} \vec{u}_r \quad (3)$$

Discretizing equation (3) we have obtained the trajectories of a trapped electron under these circumstances (figure 3).

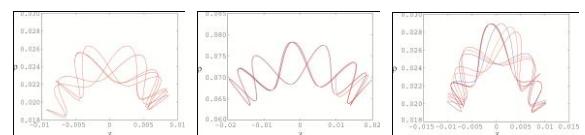


Figure 3: Trajectories of a trapped electron, in blue without the gravity effect and in red under this effect.

To get more realistic results, we have used magnetic field values obtained from the AIS ionograms radar of the MARSIS sounder at the Mars Express mission (figure 4).

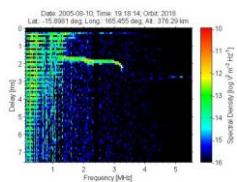


Figure 4: Ionogram obtained with the MARSIS radar.

Correcting the magnetic field according to the altitude of the spacecraft, these values have been used to describe the behavior of charged particles.

### 3.1 Störmer Theory

Although we are able to know the behaviour of charged particles and the energy they have, it is also interesting try to find the upper energy limits for an electron and a proton to get trapped in a potential well. Using Störmer theory it is possible to establish 1/32 as the minimum energy for a particle to get untrapped. In our case we have worked with Gaussian units and, in order to recover that limit, it makes necessary to work with potential (4), and search the maximum and minimum values of such function (figure 5).

$$V(\rho, z) = \frac{1}{2m} \left( \frac{p_\phi}{\rho} - \frac{e}{c} \frac{C_m m_z \rho}{(\rho^2 + z^2)^{3/2}} \right)^2 \quad (4)$$

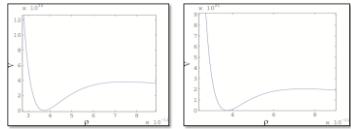


Figure 5: Potential wells for an electron (left) and a proton (right) in Gaussian units.

With these results it has been possible to find a conversion factor (5), between the dimensionless Störmer equations ( $V_s$ ) and the Gaussian equations used in this study ( $V$ ).

$$a = \frac{eC_m m_z}{p_\phi c} \quad A = \frac{ma^2}{p_\phi^2} \quad (5)$$

$$V = AV_s = A \frac{1}{2m} \left( \frac{p_\phi}{a} \right)^2 \left( \frac{1}{\rho_u} - \frac{1}{\rho_u^2} \right)$$

## 4. Summary and Conclusions

It has been possible get trapped trajectories in different situations. For an electron it is easiest to get trapped in a magnetic field similar to the one generated by a sphere for initial conditions smaller than in the case of a cylinder or for the sum of both. In the case of a proton it is the opposite situation.

The effect of a constant gravity field, it is bigger in the case of an electron. For an electron under a gravity field which varies with altitude, when the particle has a bigger velocity, this effect is bigger, but it is smaller when the particle has a high altitude (as it was supposed to be).

By using Gaussian units it has been possible to find the bounding energy limit for an electron or a proton starting from the Störmer theory. The difference between the maximum value and the minimum it is exactly twice as it was described by Störmer theory.

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