

# Interior models of Mercury with equatorial ellipticity

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## Abstract

The combination of planetary rotation observations and gravity field measurements by the MESSENGER spacecraft can be used to constrain the internal structure of Mercury. A recently published model suggests a mean mantle density of  $\rho_m = 3650 \pm 225 \text{ kg m}^{-3}$ , substantially larger than that expected of a silicate mantle ( $3300 \text{ kg m}^{-3}$ ) and possibly hinting at the presence of an FeS-rich layer at the base of the mantle. Here, we show that a large  $\rho_m$  is only required if the core-mantle boundary (CMB) of the planet is assumed axially-symmetric. An equatorial ellipticity of CMB of the order of  $2 \cdot 10^{-5}$  allows to satisfy gravity and rotation constraints with a mean mantle density typical of silicate material. Possible origin of such topography include past mantle convection, aspherical planetary shrinking, remnant tidal deformation, or a combination thereof.

## 1. Bulk density models of Mercury

We model Mercury as a triaxial planet comprised of an inner core, fluid outer core, mantle and crust, each of uniform density. The densities of the crust and inner core (pure Iron) are taken as  $\rho_c = 3100 \text{ kg m}^{-3}$  and  $\rho_s = 8160 \text{ kg m}^{-3}$ . The densities of the fluid core  $\rho_f$  and mantle  $\rho_m$  and the radius of each region must be consistent with a mean density  $\bar{\rho} = 5430 \text{ kg m}^{-3}$  and  $C/MR^2 = 0.353 \pm 0.017$  [2], where  $C$  is the polar moment of inertia,  $M$  is the total mass and  $R$  is the planetary radius. The constraint on  $C/MR^2$  is determined from observations of the degree 2 gravity field by MESSENGER and of the spin parameters under the assumption that Mercury occupies a Cassini state.

Fig. 1a shows how  $\rho_m$  changes as a function of the CMB radius ( $r_f$ ), for an assumed crustal thickness of 65 km, and for 4 choices of inner core radius. Fig. 1b shows the prediction of  $C_m/C$ , where  $C_m$  is the polar moment of inertia of the combined mantle and crust. Assuming that the CMB and ICB are both

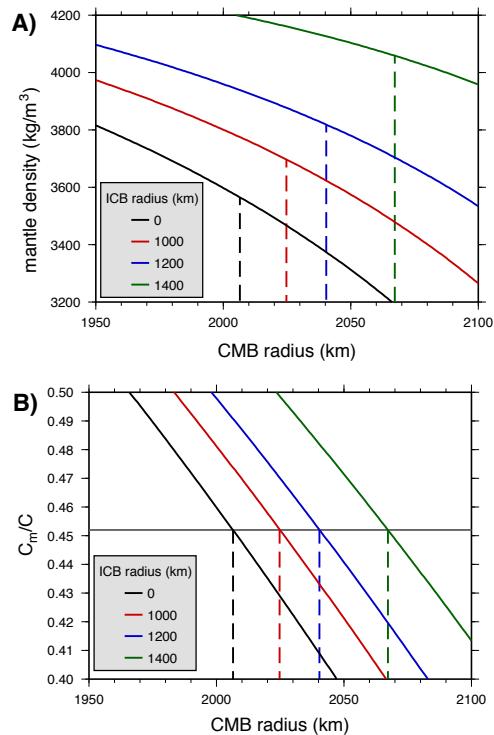


Figure 1: (A)  $\rho_m$  and (B)  $C_m/C$  as a function of CMB radius and for different choices of ICB radius. The dashed vertical lines give the values of  $\rho_m$  and  $r_f$  that match the constraint  $C_m/C = 0.452$  built from Eq. (1) (grey horizontal line on (B).)

axially-symmetric, we can write [1]

$$\frac{C_m}{C} = \frac{4C_{22}}{\Delta_m} \left( \frac{Mr_m^2}{C} \right) \quad (1)$$

where  $C_{22} = 0.81 \pm 0.01$  [2] is the (unnormalized) degree 2 equatorial variations in gravity and  $\Delta_m = (B_m - A_m)/C_m = (2.03 \pm 0.12) \cdot 10^{-4}$  is known from the amplitude of Mercury's 88-day longitudinal libration [1]. These lead to a constraint of  $C_m/C = 0.452 \pm 0.035$  [2]. Fig. 1 shows that in the absence of an inner core, the CMB radius must be  $2007 \pm 30$  km and the mean mantle density must be  $3565 \pm 140$  kg m<sup>-3</sup>.

$\text{kg m}^{-3}$ , similar to the results obtained in [2]. These differences are attributable to our specific choice of  $\rho_s$  and our assumption of uniform densities.

## 2. The role of CMB ellipticity

When the constraint of  $C_m/C$  as inferred from Eq. (1) is used, one makes an implicit assumption that the CMB and ICB are axially-symmetric. If instead we allow for non-zero equatorial ellipticity  $\beta_i$  at each of the region boundaries ( $\eta_i = r_i/R$ ), then

$$C_{22} = \frac{B - A}{4MR^2} = \sum_i \frac{\rho_i}{10\bar{\rho}} [\eta_i^5 \beta_i - \eta_{i-1}^5 \beta_{i-1}] \quad (2)$$

$$\Delta I_m = \frac{\rho_c \beta_c + (\rho_m - \rho_c) \eta_m^5 \beta_m - \rho_m \eta_f^5 \beta_f}{\rho_c (1 - \eta_m^5) + \rho_m (\eta_m^5 - \eta_f^5)} \quad (3)$$

We use the observed ellipticity of the planet along the axis of minimum inertia ( $\beta_c = [514 \cdot 10^{-6}] \cdot \cos(18.6^\circ)$ , [4]) and the condition that the ICB is at hydrostatic equilibrium, in which case its ellipticity can be written in terms of  $\beta_c$ ,  $\beta_m$  (crust-mantle) and  $\beta_f$  (CMB) [3]. Eqs. (2-3) allow us to find unique solutions for  $\beta_m$  and  $\beta_f$  that match observed values of both  $C_{22}$  and  $\Delta I_m$ . We make one additional model assumption, that the density contrast at the crust-mantle boundary in Eqs. (2-3) is  $(\rho_m - \rho_c) = 200 \text{ kg m}^{-3}$  (though we leave  $\rho_m$  to be determined by matching  $\bar{\rho}$  and  $C/MR^2$ ).

We proceed as before: for a given choice of ICB radius, we vary the CMB radius and find the combination of  $\rho_m$  and  $\rho_f$  that satisfy  $\bar{\rho}$  and  $C/MR^2$ . We then find the ellipticities from Eqs. (2-3). Fig. 2 shows how  $\beta_f$  must adjust as a function of  $\rho_m$  to be compatible with both  $C_{22}$  and  $\Delta I_m$ .  $\beta_f = 0$  correspond to the results presented in Fig. 1, for which the observed  $C_{22}$  and  $\Delta I_m$  must be explained by near surface ellipticity alone ( $\beta_c$  and  $\beta_m$ ). Fig. 2 shows how allowing for CMB ellipticity permits to explain all observational constraints without requiring a large mantle density, even for a large inner core. For instance, if  $\rho_m$  is chosen as  $3300 \text{ kg m}^{-3}$ , typical of silicate material, the required  $\beta_f$  for no inner core is  $(1.91 \pm 0.78) \cdot 10^{-5}$ . The CMB radius for this case is 2051 km. The corresponding results for ICB radii of 1000, 1200 and 1400 km are  $\beta_f = (2.81 \pm 0.68)$ ,  $(3.65 \pm 0.60)$  and  $(5.28 \pm 0.44)$  (all multiplied by  $10^{-5}$ ) with CMB radii of 2095, 2135 and 2213 km. These values depend on our choice of  $\rho_s$  and our assumption of uniform density layers. Nevertheless, our simple model illustrate the importance of considering CMB topography when constructing interior models of Mercury based on rotational and gravity observations.

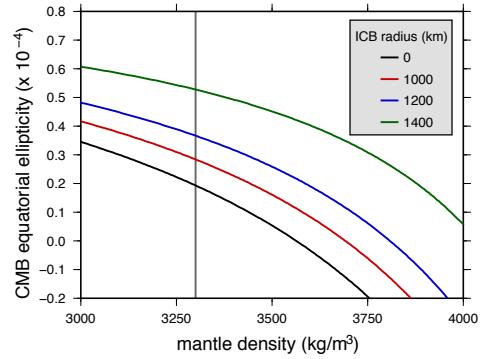


Figure 2: Equatorial ellipticity of the CMB ( $\beta_f$ ) as a function of mantle density. The vertical grey line correspond to a silicate mantle density of  $3300 \text{ kg m}^{-3}$

## 3. Summary and Conclusions

A scenario whereby a FeS-rich solid layer reside at the base of the mantle as been suggested in [2]. We have shown that this requirement disappears if the equatorial ellipticity of the CMB and ICB is taken into account. With a CMB equatorial ellipticity of the order of  $2 \cdot 10^{-5}$ , one can fit the constraints on  $\bar{\rho}$ ,  $C/MR^2$ ,  $C_{22}$  and  $\Delta I_m$  with a silicate mantle density of  $3300 \text{ kg m}^{-3}$ .

## Acknowledgements

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## References

- [1] Margot, J.-L. et al., Large longitude libration of Mercury reveals a molten core, *Science*, 316, pp. 710-714, 2007.
- [2] Smith D. E. et al., Gravity Field and Internal Structure of Mercury from MESSENGER, *Science*, 336, pp. 214-217, 2012.
- [3] Veasey, M. and Dumbrerry, M., The influence of Mercury's inner core on its physical libration, *Icarus*, 214, pp. 265-274, 2011.
- [4] Zuber, M. T. et al., Topography of the Northern Hemisphere of Mercury from MESSENGER Laser Altimetry, *Science*, 336, pp. 217-220, 2012.