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Monitoring of the Moon: unstable motion and regular orbits of lunar artificial satellites

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Abstract

In the model of the 4-body problem: the Moon and 3 artificial station - the stable trajectories for the chaotic motion and unstable orbits for the regular motion of the point with negligible mass are found.

1. Introduction

In the known cosmic programs of USA and Russia the Moon and Mars are considered as practically important celestial bodies for the earth civilization. For example, in accordance with these programs, we soon shall see habitable lunar orbital stations, developed industry on the surface of the Moon, commercial travelling at the Moon in the spaceship "Souz" in 2015.

In XXI century in celestial mechanics – in the frame of *N*-body problem - new regular trajectories of small bodies were revealed. These trajectories near the Moon, satellites and planets may be used for solving as theoretical as well practical problems of modern cosmonautics. It should be noted the simple analytical models of motion of these celestial mechanical systems are not found [1]. The authors of work [2] and [3] suggest using various models of central configurations in order to describe satisfactory these dynamical systems. Below we shall consider the motion of point with negligible mass m_0 placed (in initial moment of time) near the point of libration *L*. (In the work [4] we carried out the motion of opposite point of libration).

1. A celestial mechanical model of motion of the two gravitating lunar space stations and a small satellite

Let's consider a *central configuration* of four bodies – the Moon, two space stations, located in vertexes of an equilateral triangle with the side of a, $m_1 > m_2 = m_3$ and an artificial "satellite" of the Moon, with zero mass of m_0, in initial moment of time placed on a straight line, connecting m_1 and the middle point, placed between m_2 and m_3 (Fig.1). The bodies with mass m_1, m_2, m_3 uniformly rotate around the axes passed through the system center of mass *C*. R_2=R_3.

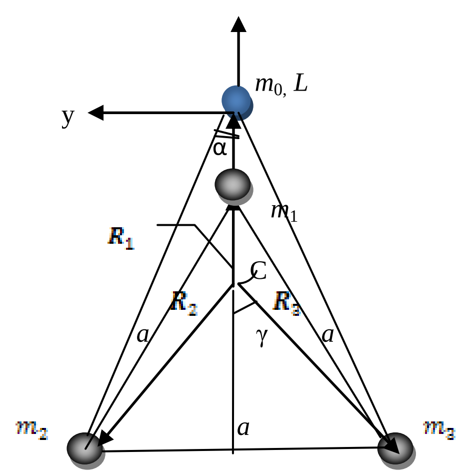


Figure 1: Four-body system: the Moon m_1 two space stations m_2 and m_3, an artificial lunar satellites m_0 (in initial moment of time m_0 coincides with point of libration *L*). *C* is the center of mass of the system, $\boldsymbol{R_1}, \boldsymbol{R_2}, \boldsymbol{R_3}$ are the radii-vectors of gravitating bodies; m_0x and m_0y are the axis chosen for considering the motion of m_0.

Taking into account only the gravitating force we have (in accordance with the work [4])

$$\omega^2 = \frac{G(m_1 + 2m_2)}{a^3} , \quad (1)$$

$$R_1 = \frac{a\sqrt{3}m_2}{m_1 + 2m_2} \quad (2)$$

$$R_2 = \frac{a\sqrt{m_1^2 + m_1 m_2 + m_2{}^2}}{m_1 + 2m_2}. \quad (3)$$

The equation of motion of the body m_0, placed in the point of libration L, in inertial system of coordinates (C*)* we present in the form

$$d^2\boldsymbol{R}/dt^2 = -\frac{Gm_1}{(R - R_1)^2 R}\boldsymbol{R} - \frac{2Gm_2}{(R^2 + R_2^2 + 2R_2 R\cos\gamma)\ R}\cos\alpha\,\boldsymbol{R}. \quad (4)$$

For the central configurations we have [2]

$$d^2\boldsymbol{R}/dt^2 = -\omega^2\boldsymbol{R}\ . \quad (5)$$

Moreover (Fig. 1),

$$\cos\gamma = \frac{m_1 R_1}{2m_2 R_2}\ , \quad (6)$$

$$\cos\alpha = \frac{R - R_1 + a\sqrt{3}/2}{(R^2 + R_2^2 + 2RR_2^2\cos\gamma)^{1/2}}\ . \quad (7)$$

Here, ω is an angular velocity of major bodies; R_1, R_2 and R_3 are the distances of the Moon and two space stations from the center of mass of the system, respectively; G is the gravitational constant; R is a distance Cm_0 for unperturbed motion of this central configuration (artificial lunar satellites m_0 is placed in the point of libration L)..

Solving the system of equations (1) – (7) we may find R for the known values m_1, $m_2 = m_3$ and a in the case of unperturbed motion.

For the perturbed motion of the body with negligible mass m_0 "near" the point L we write the corresponding equations of motion in non-inertial – uniformly rotating with angular velocity ω - system of coordinates *Lxy* in the form

$$\ddot{x} = -\frac{Gm_1(R - R_1 + x)}{((R - R_1 + x)^2 + y^2 + z^2)^{3/2}} - \frac{Gm_2(R + R_2\cos\gamma + x)}{((R + R_2\cos\gamma + x)^2 + (-R_2\sin\gamma + y)^2 + z^2)^{3/2}} -$$

$$-\frac{Gm_3(R + R_3\cos\gamma + x)}{((R + R_3\cos\gamma + x)^2 + (R_3\sin\gamma + y)^2 + z^2)^{3/2}}$$
$$2\omega\dot{y} + \varpi^2(R + x)\ ,$$

$$\ddot{y} = -\frac{Gm_1 y}{((R - R_1 + x)^2 + y^2 + z^2)^{3/2}} - \frac{Gm_2(-R_2\sin\gamma + y)}{((R + R_2\cos\gamma + x)^2 + (-R_2\sin\gamma + y)^2 + z^2)^{3/2}} -$$

$$-\frac{Gm_3(R_3\sin\gamma + y)}{((R + R_3\cos\gamma + x)^2 + (R_3\sin\gamma + y)^2 + z^2)^{3/2}} -$$
$$-2\omega\dot{x} + \varpi^2 y,$$

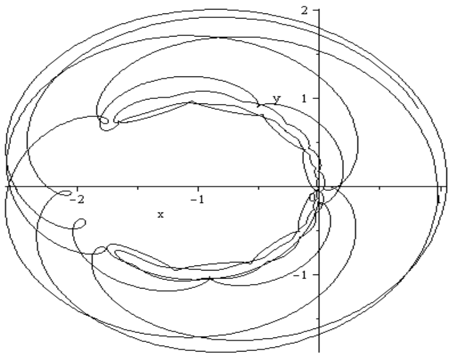
$$\ddot{z} = -\frac{Gm_1 z}{((R - R_1 + x)^2 + y^2 + z^2)^{3/2}} - \frac{Gm_2 z}{((R + R_2\cos\gamma + x)^2 + (-R_2\sin\gamma + y)^2 + z^2)^{3/2}} -$$

$$-\frac{Gm_3 z}{((R + R_3\cos\gamma + x)^2 + (R_3\sin\gamma + y)^2 + z^2)^{3/2}}$$

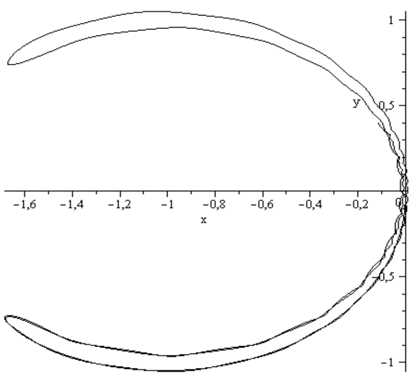
2. Examples

Consider the plane (z=0) perturbed motion of the body m_0 *in* the rotating uniformly with angular velocity ω system of coordinates *Lxy*. We put $m_2 = m_3 = 1$, G=1, a=1, .$x(0)=y(0)=0$; $(dx/dt)_0 = (dy/dt)_0 = 10^{-9}$ units of velocity; $y_2 = -y_3 = 0.5$; intervals of time are equal to 10-2000 units of time. x_2, y_2 and x_3, y_3 are the coordinates of the major bodies with mass m_2 and m_3 in the system *Lxy*. n is a number of revolutions of the major bodies. Method of Runge-Kutta (fourth order version, 24 signs) gives the following results

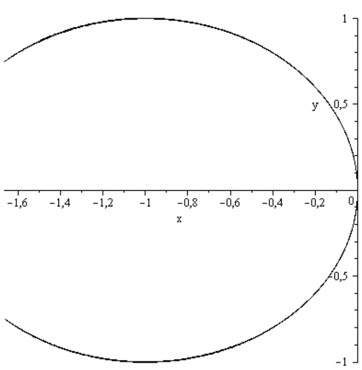
For m_1/m_2 =1100, we see “chaotic” motion of m_0 (Fig. 2a).
For m_1/m_2 =1500 we see “regular” motion of $m_{0.}$ in restricted region (Fig. 2b).
For m_1/m_2 =5000 and especially for m_1/m_2 =1000000, we see regular motion of m_0 in significantly restricted regions, like arc of a ring (Fig. 2c, 2d). (The point of libration *L* is not stable - in accordance with Lyapunov theorems).



a)



b)



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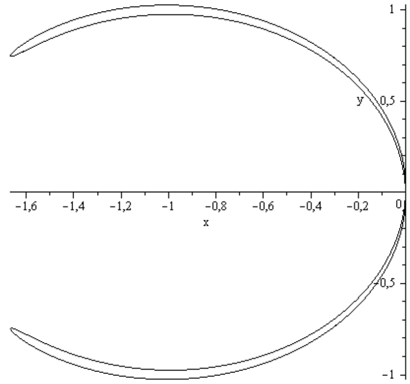

c)

Figure 2: Trajectories of the lunar artificial satellites m_0 in the gravitational fields of the Moon and two lunar space stations. *a*) m_1/m_2=1100, ω^2=1102, R/a = 1.0005993527, R_1 = 0.0015717339, R_2 = R_3 = 0.9986391476, x_2=x_3=-1.8650530225, n = 105.667376; *b*) m_1/m_2 =1500, ω^2=1502, R/a = 1.0004397547, R_1 = 0.0011531629, R_2 = R_3 = 0.9990014979, x_2=x_3=-1.8653119955, n = 123.36304869400; *c*) m_1/m_2=5000, ω^2=5002, R/a =1.000132059061, R_1 =0.00034627165285, R_2 = R_3 =0.99970013494452, x_2 = x_3= -1.865811191193, n =225.12409035; *d*) m_1/m_2=1000000, ω^2=1000002, R/a = 1.000000660578, R_1 =0.000001732047, R_2 = R_3 =0.999998500003, x_2=x_3=-1.86602433231557, n =7161.979601104.

Conclusion

In work [5] of Perov N. I. and Medvedev Yu. D. it is stressed, system, similar considered above the central configuration of four bodies (Fig. 1) is stable due criterion of Luapunov at the ratio of mass of components m_1/m_2=m_1/m_3>367.0540108.This value is greater in comparison with mass ratio (m_1/m_2~25) needed for the stability of the system of the major three bodies. So, it follows, from the considered model, that rings of (small) artificial satellites with stable orbits should be made at appreciable distinction of mass of the celestial body and the satellite (m_1/m_2>1000).

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References

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