

## Power laws and classification of planets

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### Abstract

The aim of this work is to use known data of the Solar System Planets and Transiting Exoplanets (specifically the radius and mass) to obtain empirical laws for the planetary radius, mean density, and surface gravity as a function of mass. The method used is to calculate with the available data, the mean density and surface gravity for the planets and adjust, using the least squares method, a function with respect to the radius-mass, density-mass and surface gravity-mass relations. The most important results of this work are: In the mass interval from  $10^{19}$  to  $10^{29}$  kg, the planets separate in a natural way into three groups or classes which I called class A, class B and class C. In all these classes and with all the variables (radius, mean density and surface gravity) the best-fitted functions are power laws.

### 1. Introduction

When studying the general laws governing the structure of bodies such as the Exoplanets, Jupiter, The Earth, The Moon and Titan, Enceladus and Pluto it is not relevant whether they are a moon, a planet or a dwarf planet. Together could be called planetary bodies, but to save words I will call at all, only planets. So in this work is taken as a planet any body that conforms to the following definition: A celestial body, no matter how it has formed, no matter which site or neighborhood is located, which has a mass that is below the threshold mass for thermonuclear fusion of deuterium, and a mass large enough so that its shape (spherical) is determined by a viscous relaxation induced by gravity.

One of the main problems, when studying a planet, is finding out how the mass is distributed within the interior [1], [2]. The first step to understanding the distribution within the interior of the planets is to see

how the radius, mean density and surface gravity behave as the planet's mass increases. The mass  $M$  and radius  $R$  of the planets in our Solar System as well as the mass and radius of the transiting extrasolar planets are known, so those parameters can be calculated. Using the data of 26 planets of Solar System [3] and 92 transiting exoplanets [4], an empirical power law  $R(M)$  was obtained for all these bodies. Planets with a mass  $> 3 \times 10^{25}$  kg do not adjust too well to this general radius-mass law and they seem to form two independent groups. For this reason the planets were separated in three classes A, B, and C. Class A is for the planets with a mass  $< 3 \times 10^{25}$  kg, Class B has the ones in the range of  $3 \times 10^{25} < M < 1 \times 10^{27}$  and Class C are the planets with a mass  $> 10^{27}$  kg. Taking separately each class, each one satisfies a different radius-mass power law. The slope in the power law of class A is less pronounced than the one of the general power law. The power law of class B has a greater slope and class C has no slope, showing that in this class the radius remains constant when the mass increases. I also got power laws for the mean density of the three classes of planets studied. Here also we see, more clearly, the separation of the planets into three classes. In classes A and C the density increases when mass increases, but with class B the density decreases when mass increases. Power laws for surface gravity were also obtained for these three groups of planets and here again clearly shows the separation into three classes.

### 2. Radius, density and surface gravity as function of mass

In figure 1 we can see the power laws for the radius, in figure 2 the power laws for the density and in figure 3 the power laws for the surface gravity, all of these as a function of mass for the three classes.

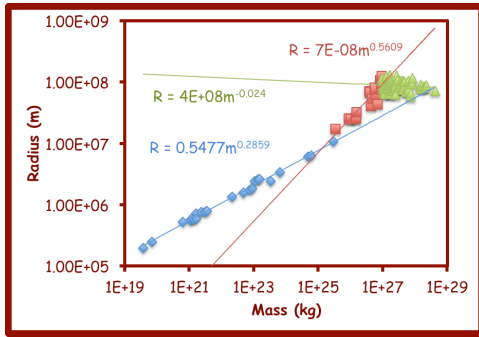


Figure 1: Radius as function of Mass for the three classes. Class A (Blue), Class B (Orange), Class C (Green).

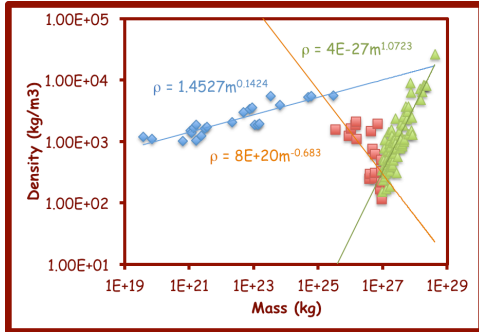


Figure 2: Density as function of Mass for the three classes.

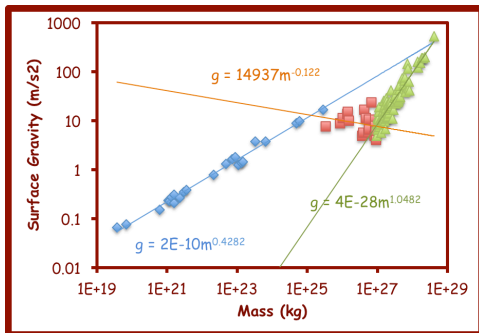


Figure 3: Surface gravity as function of Mass for the three classes.

### 3 Summary and Conclusions

The most relevant conclusions of this work are:

- 1 The planets radius as well as the mean planet density and the surface gravity are power laws of mass.
- 2 The planets, in the mass interval from  $10^{19}$  to  $10^{29}$ kg, are clearly separated into three groups or classes of planets.
- 3 The A-class planets, which Earth and nearly all the planets of the Solar System belong, are in the mass interval of  $10^{19} < M < 3 \times 10^{25}$ kg, and the radius, density and surface gravity increase when the mass increases.
- 4 The B-class planets, which Saturn, Uranus and Neptune belong, are within the mass interval  $3 \times 10^{25}$ kg  $< M < 10^{27}$ kg. The radius increases with the mass and have the unexpected property of decreasing their density and their surface gravity when increasing their mass.
- 5 The C-class planets, to which Jupiter belongs, are in the mass interval  $10^{27} < M < 10^{29}$ kg and have the property that their radiuses stay constant while increasing their mass.

### References

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