

# Rotational dynamics of a viscoelastic Europa

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## Abstract

Jupiter's moon Europa is expected to have a global subsurface ocean sandwiched between an outer ice shell and a rocky mantle. Due to the presence of this liquid layer, differential rotation between the rocky interior and the shell may occur. The rotation of both groups of layers, although mechanically decoupled, remains coupled through internal gravitational and pressure torques. In this study, we aim to present an analysis of the rotational dynamics of Europa from a (visco)-elastic point of view by solving the coupled Liouville equations for the ice shell and the rocky mantle. In addition, we redefine the gravitational and pressure torques in order to take into account the effect of viscoelasticity on the internal coupling between layers.

## 1. Introduction

In the plausible case that a global subsurface ocean is present in the interior of Europa, the icy shell could experience differential rotation with respect to the deep interior. Although the shell may be mechanically decoupled from the interior by the presence of an ocean, the rotation of the shell remains coupled to the interior by the work of gravitational and pressure torques. The influence of these coupling torques on the rotation of the shell has been extensively studied for the case in which the shell behaves as a rigid body [1, 2, 3]. In these models, the effect of (visco)-elasticity on the rotational behavior of the shell and the rocky interior has been neglected. Nevertheless, viscoelastic relaxation is expected to reduce the magnitude of the coupling torques as the shell is able to adapt its shape faster to the one dictated by the forcing field [4, 5].

## 2. Rotational Dynamics

We consider the interior of Europa to consist of 4 layers of homogeneous density: fluid core, rocky mantle, ocean and ice shell. In order to take into account the effect of (visco)-elasticity into the modeling, we treat the rocky mantle and the ice shell as viscoelastic layers. In agreement with thermal and impact models, we subdivide the ice shell into two rheological regimes: an upper nearly-elastic thin layer and a larger lower ductile layer. We then compute the time-dependent Love numbers  $h_2(t)$ ,  $l_2(t)$  and  $k_2(t)$  and the corresponding elastic and fluid Love numbers at every layer interface by means of the method explained in [6].

The presence of a subsurface ocean between the ice shell and the rocky mantle decouples the rotation of these two layers, at least from a mechanical viewpoint. The rotation of both layers remains, however, coupled by the action of gravitational and pressure torques. Then, the linearized Liouville equations for the ice shell can be written as:

$$A_s \Omega \dot{m}_x^s + (C_s - B_s) \Omega^2 (1 - \zeta^s) * m_y^s = \Gamma_x^s, (1)$$

$$B_s \Omega \dot{m}_y^s - (C_s - A_s) \Omega^2 (1 - \zeta^s) * m_x^s = \Gamma_y^s, (2)$$

$$C_s \Omega \dot{m}_z^s + \frac{4}{9} (B_s - A_s) \Omega (1 - \zeta^s) * \dot{m}_z^s = \Gamma_z^s, (3)$$

where  $A_s$ ,  $B_s$  and  $C_s$  are the principal moments of inertia of the shell;  $m_x^s$ ,  $m_y^s$  and  $m_z^s$  are the rotational variations of the shell;  $\Omega$  is the magnitude of the Europa's angular velocity;  $\Gamma_x^s$ ,  $\Gamma_y^s$  and  $\Gamma_z^s$  are the components of the torque applied on the shell (including gravitational and pressure torques); and the ratio  $\zeta^s$  is given by

$$\zeta^s = \frac{h_2(r_s) - h_2(r_o)\tau^6}{h_f(r_s) - h_f(r_o)\tau^6}, (4)$$

with  $r_s$  and  $r_o$  being the outer radius of the shell and the outer radius of the ocean, respectively. The parameter  $\tau$  is defined as the ratio  $r_o/r_s$ .

The rotational state of the shell can then be described by the solutions of equations 1 to 3. In reality, these equations are coupled to the Liouville equations of the rocky mantle, which are similar to equations 1 to 3 but

with a subscript/superscript  $m$  rather than  $s$ , through gravitational and pressure torques.

### 3. Coupling Torques

(Visco-)elasticity does not only affect the rotational dynamics of the shell through the adjustment of the equatorial and tidal bulges, but also through the coupling torques acting on the shell. For example, the gravitational torque of the interior on the shell in the  $z$ -direction,  $\Gamma_{z,g}^s$ , is given by

$$\Gamma_{z,g}^s = K_s [\varepsilon^s * 2\alpha_z^s - \zeta^I * 2\alpha_z^m], \quad (5)$$

where

$$K_s = -\frac{4\pi G}{5} (B_I - A_I) \rho_s [\beta(r_s) - \beta(r_o)], \quad (6)$$

$$\varepsilon^s = \frac{h_2(r_s) - h_2(r_o)\tau}{h_f(r_s) - h_f(r_o)\tau}, \quad (7)$$

$$\begin{aligned} \zeta^I = & \left\{ -\rho_o h_2(r_m) + \rho_m [h_2(r_m) - h_2(r_c)\delta^6] + \right. \\ & \left. \rho_c h_2(r_c)\delta^6 \right\} \cdot \left\{ -\rho_o h_f(r_m) \right. \\ & \left. + \rho_m [h_f(r_m) - h_f(r_c)\delta^6] + \rho_c h_f(r_c)\delta^6 \right\}^{-1}, \end{aligned} \quad (8)$$

with  $\rho_s$ ,  $\rho_o$ ,  $\rho_m$  and  $\rho_c$  being the density of the ice shell, ocean, rocky mantle and liquid core, respectively. Furthermore,  $A_I$  and  $B_I$  are the equatorial principal moments of inertia of the interior;  $\beta(r)$  is the equatorial flattening;  $\delta$  is the ratio  $r_c/r_m$ ; and  $\alpha_z^s$  and  $\alpha_z^m$  represent the rotation angle of the shell and the mantle, respectively.

A close inspection of the ratios  $\varepsilon^s$  and  $\zeta^I$  shows that in most cases  $\varepsilon^s$  will be considerably larger than  $\zeta^I$  as a result of the large elastic component in the deformation of the shell and faster viscoelastic relaxation of the shell relative to the rocky mantle. Consequently, the gravitational torque of the interior on the ice shell will mainly depend on the rotation of the shell for timescales shorter than the Maxwell time of the rocky mantle. In the long-term, both ratios will approach unity as the Love numbers will approach the fluid limit. In that case, the gravitational torque given by equation 5 simplifies to the commonly known expression for rigid bodies, as given in e.g. [1, 2, 3, 4]. Similar expressions can be derived for the pressure torques and the other components of the gravitational torque.

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