

# Tidal dissipation in the Moon. Learning from the “incorrect” frequency dependence measured by the LLR

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## Abstract

It was demonstrated back in 2001 that fitting of the LLR data results in the quality factor  $Q$  of the Moon scaling as the frequency  $\chi$  to a negative power [8]:

$$Q \sim \chi^p, \quad \text{where } p = -0.19. \quad (1)$$

At the same time, numerous measurements by various seismological teams agree on the exponent being positive, not negative [4]. The positive sign of the exponent stems also from geodetic measurements [1], and it finds its explanation within the theory of friction in minerals [5]. On all these grounds, the aforementioned finding by the LLR team appears to be implausible and to disagree with the conventional wisdom of solid state mechanics and seismology.

A later reexamination in [9] rendered a less upsetting value,  $p = -0.09$ , which was still negative and still seemed to contradict our knowledge of microphysical processes in solids. The authors later wrote [10]:

*“There is a weak dependence of tidal specific dissipation  $Q$  on period. The  $Q$  increases from  $\sim 30$  at a month to  $\sim 35$  at one year.  $Q$  for rock is expected to have a weak dependence on tidal period, but it is expected to decrease with period rather than increase. The frequency dependence of  $Q$  deserves further attention and should be improved.”*

A possible explanation of this paradox comes from the observation that the LLR measurements provided information on the *tidal* and not seismic dissipation. The difference between these two processes comes from self-gravitation of the celestial body. To address the problem accurately, one has to calculate the tidal factors  $k_l \sin \epsilon_l$  showing up in the Darwin-Kaula expansion for the tidal torque or force. Here  $k_l$  is the degree- $l$  Love number, while  $\epsilon_l$  is the appropriate tidal lag. Sometimes  $\sin \epsilon_l$  is denoted with  $1/Q$ , which is not recommended, because this notation does

not distinguish between the tidal reaction appropriate to harmonics of different degree. This notation also puts one at risk of confusing the tidal damping with the seismic damping, two processes that have much in common but are nevertheless different [2, 3]. The factors  $k_l \sin \epsilon_l$  are functions of the tidal modes  $\omega_{lmpq}$ , where  $lmpq$  are integers used to number the modes. (The tidal modes can be either positive or negative, while the appropriate tidal forcing frequencies in the mantle,  $\chi_{lmpq} = |\omega_{lmpq}|$ , are positively defined.) So the  $lmpq$  term in the expansion of tide is proportional to  $k_l(\omega_{lmpq}) \sin \epsilon_l(\omega_{lmpq})$ .

An accurate calculation demonstrates that for realistic rheologies the tidal factors  $k_l \sin \epsilon_l$  have a maximum at a frequency, which is (for not too large bodies) about the inverse Maxwell time [2, 3]. In the zero-frequency limit, the factors go smoothly through nil and change their sign, a natural behaviour saving the theory from an infinite torque or force at a resonance crossing.

As the small negative exponent was derived from LLR observations over periods of a month to a year, we see that the appropriate frequencies were close to or slightly below the frequency at which the factor  $k_2 \sin \epsilon_2$  has its peak. Taken that the said frequency is not very different from the inverse Maxwell time, we estimate the typical viscosity  $\eta$  of the Lunar mantle as<sup>2</sup>

$$\eta = 3 \times 10^{15} \text{ Pa s}. \quad (2)$$

Such a low viscosity may indicate that the lower lunar mantle contains a high percentage of partial melt. This interpretation goes along with the model developed in [6] and advocated later in [8] and [10]. It also agrees with the recent model offered in [7].

<sup>2</sup> Mind a misprint in the value of  $\eta$  provided in our papers [2] and [3].

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