

# Accurate database for constrained linear unmixing

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## Abstract

Detecting minerals on a very large hyperspectral dataset ( $> 10^6$  pixels) is a difficult task that may be solved using fast linear unmixing techniques under constraints of positivity and sum-to-one. We test different algorithms and different reference database on typical Martian hyperspectral images of the Syrtis Major volcanic complex from OMEGA and CRISM. We show that new algorithms can handle linearly dependent spectra in the database as expected theoretically. This possibility offers a new opportunity to fit the continuum spectra.

## 1. Introduction

In remote sensing hyperspectral imaging, a set of images is recorded at various spectral bands by the sensor which measures the solar light reflected and scattered back from the surface and from the atmosphere. Under some assumptions related to surface and atmosphere properties, each measured spectrum (each pixel of the observed image for several spectral bands) can be modeled as a linear mixture of the scene component spectra (endmembers). Using matrix notations, one can write:

$$\mathbf{X} \approx \mathbf{AS} \quad (1)$$

where each row of  $\mathbf{X}$  contains the  $p$ -th pixel spectrum and matrix  $\mathbf{S}$  contains the endmember spectra. In this model, the weight  $\mathbf{A}$  of each component spectrum  $\mathbf{S}$  is related to its abundance in the surface area corresponding to the underlying pixel. Supervised linear unmixing problem consists of estimating matrix  $\mathbf{A}$  knowing  $\mathbf{X}$  and  $\mathbf{S}$ , in contrary to unsupervised unmixing that consists of estimating matrix  $\mathbf{A}$  and  $\mathbf{S}$ , knowing only  $\mathbf{X}$  [1]. A first hard constraint is the non-negativity of the elements of  $\mathbf{A}$  since they correspond to abundances of the surface components:

$$A_{p,r} \geq 0, \forall p, r \quad (2)$$

A second constraint that may be imposed is the sum-to-one (additivity) constraint on the abundances that should sum to unity for each pixel  $p$ :

$$\sum_r A_{p,r} = 1, \forall p \quad (3)$$

Thanks to both constraints (2) and (3), the problem is not undetermined using some linearly mixed spectra  $\mathbf{S}$ . We tested here the behavior of two algorithms to fit the continuum using this possibility.

## 2. Methods

We select the observation OMEGA observation ORB0422\_4 and the CRISM observation frt0000a09c of Syrtis Major as a test case because this single cube contains well-identified areas with very strong signatures of mafic minerals (orthopyroxene, clinopyroxene, olivine) and phyllosilicates. The images have been radiometrically corrected and the atmospheric gas transmission has been empirically corrected using the volcano scan method.

The FCLS method solves the unmixing problem under non-negativity and sum-to-one constraints [2]. Since no closed form expression of the optimal abundance vector can be derived under these two constraints, an iterative scheme is developed. The non-negativity constraint is classically handled by introducing the Lagrange function associated with the criterion to be optimized. The sum-to-one constraint is considered as an additional measurement equation leading to a new cost function. This method was previously tested to detect ices and liquid water on Mars [3]. An alternative approach, called IP-FCLS was recently developed [4] using primal-dual interior point optimization approach.

For both algorithms, we tested different reference spectral database  $\mathbf{S}$ , including several linearly dependent data.

### 3. Results

Figure 1 present the collection of 32 reference spectra from laboratory measurements and synthetic data [5] that from our expertise is the best compromise to fit the data using FCLS. It corresponds to the maximum linearly dependent spectra allowed in the algorithm. If one adds the positive slope for instance, the matrix inversion of the FCLS algorithm fails.

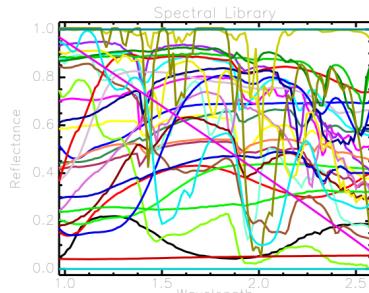


Figure 1: Reference database of 32 spectra, including two flat at 0.01 and at 1.0 and the negative slope.

This behavior is not present for the IP-FCLS algorithm since no matrix inversion is involved. One can add many linearly dependant spectra, opening the possibility to better fit the continuum and tackle the non-linearity due to complex radiative transfer in the surface and atmosphere. In practice, the most reasonable additional artificial spectra are a basis of sine and cosine at very large wavelength in order to fit the general shape of the continuum. We tested only 4x and 2x period in order to prevent from specific absorption bands of some minerals, like pyroxenes (figure 2 represent the set of artificial spectra).

Figure 3 show an example of a fit with successful continuum estimation. The results show a detection of four minerals out of 29 (sparse results) with a very dominant one (clay: illite).

### 4. Summary and conclusion

We showed that recent algorithms of linear unmixing including positivity and sum-to-one constraints could provide new possibility to fast fit the continuum in addition to “spectral abundance” estimation.

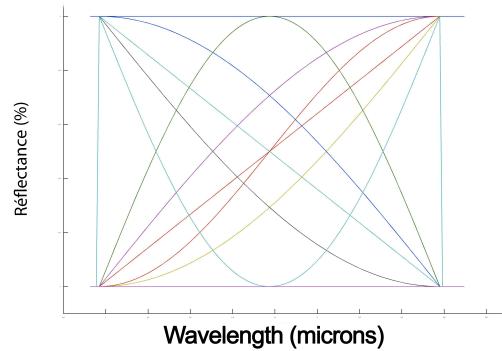


Figure 2: Additional artificial database of positive slope, sine and cosine with different phase shift.

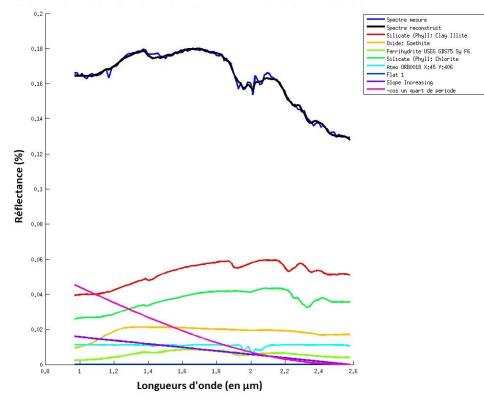


Figure 3: Example of a fit with 41 spectra in the database (most complete one).

### References

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