

# Radiative transfer inversion for hyperspectral data set on Mars

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## Abstract

**Time evolution of CO<sub>2</sub> ice state and composition are keys to constrain the physical behavior of Martian seasonal processes. Here we present a radiative transfer inversion method based on maximum likelihood to retrieve quantitatively these properties. Our method separates the direct model and the comparison between the data and the direct model by creating a look up table, to reduce the numerical cost. Still, it gives an estimation on the quality of the sampling and the uncertainty on the solution.**

## 1. Introduction

The purpose of this work is to build an inversion method rapid enough to allow the inversion of a large amount of spectra, to retrieve the Martian surface characteristics : CO<sub>2</sub> ice state and thickness, proportions and grain sizes of impurities. Since (i) the radiative transfer equations [1,2] cannot be inverted analytically, (ii) a Monte Carlo method [4] is too slow due to the high spectral resolution of the ice bands (one direct computation of a spectra of 54000 sampling at 0,5 cm<sup>-1</sup> takes 0,7 s), (iii) the number of spectra to invert is very high (up to 100 000 spectra per image) we built a method based on a spectral library database, and comparison between the measured data an the spectral library.

## 2. Method

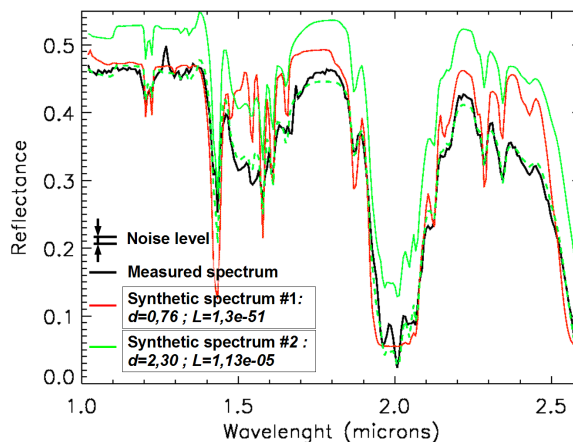
The first step of the method is to generate a spectral library, using a radiative transfer model [2] at high resolution, down sampled at the instrument resolution of CRISM [5] (computation time ~15 days). This library will contain different synthetic spectra, sampling regularly the parameters space. Once this library is created, it is stored as a look up table. The second step is to compare the measured data of CRISM [5], corrected from the atmosphere

contribution (gas and aerosols) [3] to the synthetic database. We developed a spectral comparison method (inversion) that allows taking into account any possible bias in atmospheric correction. It consists in a likelihood function defined as follows :

$$L = \exp \left( -\frac{1}{2} \times {}^t (d_{sim} - d_{mes}) \overline{C}^{-1} (d_{sim} - d_{mes}) \right)$$

where  $d_{sim}$  and  $d_{mes}$  are the simulated and measured spectra, and  $\overline{C}$  is a  $n_b \times n_b$  covariance matrix,  $n_b$  being the number of spectral bands used.

The solution is the marginal density of the likelihood for each parameter (see fig. 2). It can be interpreted as an *a posteriori* state of information [4] if the *a priori* information is constant in the parameter space.



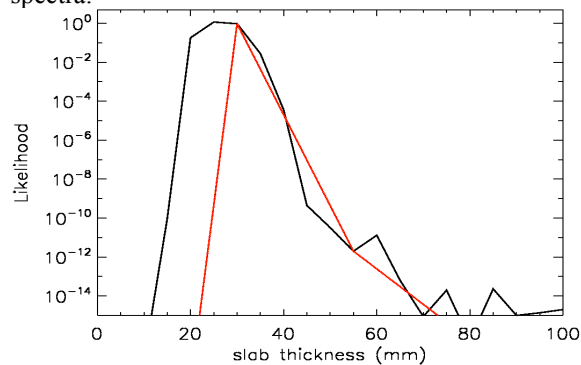
**Figure 1** : Illustration of the interest of the covariance matrix. The red and green synthetic spectra are compared to the measured one (black),  $d$  being the distance given by the least squares method, and  $L$  the likelihood given by our method. The green dashed spectrum corresponds to the green one from which we have withdrawn the components on the first two eigenvectors of  $\overline{C}$  (level and slope). According to the least squares method, the red spectrum has a better agreement with the measure, its distance being smaller, but our method chooses the green one, its likelihood being many order of magnitude bigger than the red's. Note that a perfect match has a likelihood  $L=1$  (and a distance  $d=0$ ).

### 3. Covariance matrix

$\bar{C}$  is the covariance matrix of the measured spectra we expect. In the bayesian framework under gaussian hypothesis, it shall represent the state of uncertainty of the data [4]. A diagonal coefficient  $C_{i,i}$  represents the variance of the reflectance at the wavelength  $i$ , and the coefficient  $C_{i,j}$  represents the covariance between the reflectances at the wavelengths  $i$  and  $j$ .

$\bar{C}$  will traduce different sources of uncertainties : (i) the measure itself (noise), (ii) the uncertainties we expect from the atmospheric corrections (aerosols optical thickness...), (iii) the uncertainties in the estimation of the geometry (incidence and emergence), due to local topography. From numerical simulation, we identify that at first order both atmospheric correction and geometry uncertainties may change the level ( $\pm 0.1$ ) and the global slope of spectra ( $\pm 0.03 \mu m^{-1}$ ). Therefore, we created the two corresponding eigenvectors/eigenvalues into  $\bar{C}$ . The noise subspace (dimension  $n_b-2$ ) has been constructed orthogonally to the previous 2D subspace using the Gram-Schmidt process.

Using such a matrix instead of a much simple least squares method with diagonal  $\bar{C}$  improves the description of the data uncertainty. Also, it removes a time consuming ad hoc fitting step of continuum and slope for each couple of observed and look up table spectra.



**Figure 2** : Preliminary result of the likelihood versus value of the slab ice thickness. (red) Poor sampling : only one value is likely to be the solution, when the likelihood of the other parameters is negligible. (black) More satisfactory sampling : the likelihood has a bell shaped around the most likely value.

### 4. Results and discussion

The calculation time depends highly on the sampling of the parameters space. The coarser the sampling is, the faster is the computation. But the precision of the

inversion is directly bound to the sampling of the parameters, so an iterative process is required until the sampling is satisfactory, that is when the uncertainties on the result do not depend on the sampling. On fig. 2 you can see a poor sampling in red and a better one in black. The sampling giving the black curve shows a good apparent sampling of the marginal density. The computation time needed for each spectrum once the library is computed is about 4 s, that allows the inversion of a large amount of spectra.

### 5. Conclusion

We built an inversion method that combines the computation speed of a lookup table and the statistical advantages of a bayesian method. The quality of the inversion depends on the quality of the sampling and the covariance matrix. Once the spectral library is computed, this method permits to compute an inversion of one measured spectrum in a couple of seconds, where a Monte Carlo would take days. Creation of the library is the long step, but only required once.

### References

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