EPSC Abstracts
Vol. 8, EPSC2013-22-1, 2013
European Planetary Science Congress 2013

Interior Models of Saturn Including the Uncertainties in Shape and Rotation

R. Helled (1), T. Guillot (2)
(1) Dept. of Geophysical, Atmospheric, and Planetary Sciences, Tel-Aviv University, Israel
(2) Université de Nice-Sophia Antipolis, Observatoire de la *Côte d'Azur*, Nice, France
rhelled@post.tau.ac.il

Abstract

The accurate determination of Saturn's gravitational coefficients by Cassini could provide tighter constrains on Saturn's internal structure. Also, occultation measurements provide important information on the planetary shape which is often not considered in structure models. In this work we explore how wind velocities and internal rotation affect the planetary shape and the constraints on Saturn's interior. Saturn's equatorial and polar radii at 100 mbar are derived to be 54,445 ±10 km and 60,365±10 km, respectively. We then determine Saturn's interior when the constraints on the shape and the uncertainty in rotation period/state are included. With Voyager's period, the derived mass of heavy elements in Saturn's envelope is 0-7 $M_\oplus$ while with a rotation period of 10h32mns, this value becomes $< 4\ M_\oplus$. Saturn's core mass is found to depend strongly on the pressure at which helium phase separation occurs, and is estimated to be 5-20 $M_\oplus$. Lower core masses are possible if the separation occurs deeper than 4 Mbars. We suggest that the analysis of Cassini's radio occultation measurements is crucial to test shape models and could lead to constraints on Saturn's rotation profile and departures from hydrostatic equilibrium. This work is based on Helled & Guillot (2013).

1. Introduction

Knowledge of giant planets' internal structures is crucial for understanding giant planet origin. Although Cassini's accurate measurements of Saturn's gravitational field offer an opportunity to better constrain Saturn's internal structure, the realization that Saturn's rotation period is not well constrained within a few minutes introduces an uncertainty that must be considered when modeling the planetary interior. The interior of a giant planet is typically derived for a given solid-body (SB) rotation period and the inferred model (i.e., composition and its depth dependence) is therefore dependent on the assumed rotation period and on the assumption of SB rotation. The uncertainty in the planetary rotation also introduces uncertainty in the planetary hydrostatic shape which enters the interior models via the mean/equatorial radius. So far interior models of Saturn assumed equatorial radius and SB rotation rate which result in an incorrect mean (volumetric) radius. In this work we suggest how the uncertainty in Saturn's rotation period/state and its corresponding shape should be treated when deriving structure models.

2. Rotation Period

It has been acknowledged that Saturn's rotation period is unknown within a few minutes since Cassini's SKR measured a rotation period longer by ~ 8 minutes than the Voyager radio period of 10h 39mns 22.4s. In addition, during Cassini's orbit this period was found to be changing with time. Due to the alignment of the magnetic pole with the rotation axis, Saturn's rotation period cannot be obtained from magnetic field measurements. Several groups have attempted to determine the rotation period of Saturn's deep interior using various methods. Anderson and Schubert (2007 -hereafter AS07) have argued that Saturn's rotation period can be found by minimizing the dynamical heights and suggested a rotation period of 10h 32mns 35s. Read et al. (2009) derived a rotation period of 10h 33mns 13s based on dynamical arguments.

3. Shape

The shape of a rotating planet in hydrostatic equilibrium (the *reference geoid*) is defined as the level-surface of an equal effective potential. The measured planetary shape, however, is also affected by atmospheric winds. The contribution of the winds to the shape, also known as the 'dynamical heights' (Lindal et al., 1985) can be added to the reference geoid shape in order to reproduce the physical shape. By using zonal wind data with respect to an assumed SB rotation period, the dynamical heights with respect to the reference geoid can be derived (Lindal et al., 1985). It's important to note that all interior models of the giant planets published thus far are *hydrostatic* and 1D.

The physical parameters that are used to constrain the planetary interior are the planet's total mass, its rotation period, the measured gravitational harmonics and the planet's equatorial/mean radius. Although Saturn's shape is determined independently of rotation period by occultations, the measured radii include the contributions from the wind and therefore should *not* be compared with the equilibrium shape of a SB rotating planet as derived by interior models. As a result, interior models should account for the uncertainty in the polar, mean, and equatorial radii of Saturn, and should not use the measured values which correspond to the physical shape of the planet.

We compute Saturn's physical shape for three different SB rotation periods: Voyager radio period of 10h 39mns 22.4, 10h 45mns 24s and AS07 period of 10h 32mns 35s using wind velocities data. The derived dynamical heights vs. latitude are shown in Fig. 1. The red, gray and black solid lines represent Voyager's period, 10h 45mns 24s, and AS07 period, respectively. Since the wind velocity data do not go all the way to the poles, at high latitudes the wind velocities are set to zero. The dynamical heights are larger for longer rotation periods, and for AS07 rotation period they are minimized.

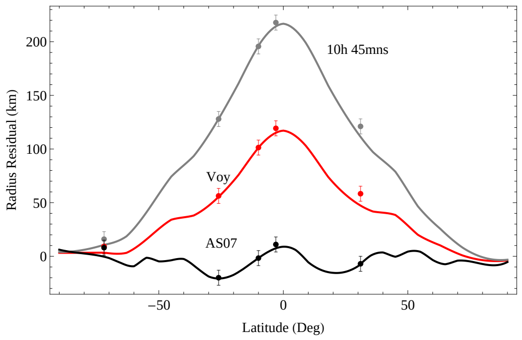

Figure 1: Saturn's dynamical heights (100 mbar) vs. altitude for rotation periods of 10h 32mns 35s (AS07, black), 10h 39mns 22.4s (Voyager, red), and 10h 45mns 24s (gray). The circles represent radii obtained by radio occultation measurements with an error of 7 km.

We also derive the radius residuals for the shape of a geoid with the Voyager rotation period and the equatorial radius reported by Lindal et al. (1985) as typically assumed by previous interior models. The residuals are found to be small near the equator but increase up to 90 km at the polar regions (not shown). The ∼100 km difference in Saturn's polar radius is not surprising given that the dynamical heights of Saturn with Voyager's period are of the same order. We therefore suggest that this combination, which was often assumed by interior modelers, is *inconsistent with the available data*. Saturn's equatorial, polar and mean radii at the 100 mbar pressure-level are found to be 60,365±10 km, 54,445 ±10 km and 58,323±10 km, respectively.

4. Interior Models

The uncertainties in Saturn's internal rotation and shape should be included in structure models as this can affect the resulting gravitational potential, and thus what we can infer from its measurement. Until today, all interior models of Saturn have been calculated using Saturn's measured equatorial radius and a SB rotation set to Voyager's system III value, which leads to Saturn models with a polar radius that is off by ∼120 km compared to the observations, and hence, to a wrong mean radius. While the effect is small, it is much larger than the observational uncertainties so that it is important to quantify it. We therefore consider three different cases: Case (0) corresponds to previous studies in which Saturn's equatorial radius was held equal to the observed value. Case (1) corresponds to a case in which the polar radius is held equal to its measured value. Finally, Case (2) corresponds to fixing the mean radius to its observed value. These cases might not represent realistic configurations but they are useful in defining the range of Saturn's equatorial/mean radius for an interior modeler. We suggest that the uncertainty in Saturn's equatorial (polar) radius for a given SB rotation period should be taken as the (absolute) *difference* between the equatorial (polar) radius in Case (1) and Case (2).

To model the planetary interior we use a standard interior model in which the planet is assumed to consist of a central ice/rock core and an envelope which is split into a helium-rich metallic hydrogen region and a helium-poor molecular region. The models are computed assuming that the 1-bar temperature ranges between 130 and 145 K, the helium mass fraction at the outer envelope ranges between 0.11 and 0.25, with the overall helium mass fraction of 0.265-0.275, and that the pressure in which the transition from the

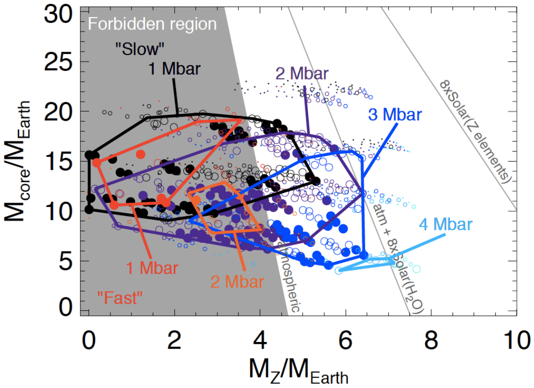

Figure 2: Saturn's core mass (M_{core}) vs. the mass of heavy elements in the envelope (M_Z) for interior models matching the available observational constraints using Cassini's Js, combining Case (0), Case (1), and Case (2): (i) Voyager rotation period ("Slow") and $P_{transition}$ = 1 (black), 2 (purple), 3 (blue), and 4 Mbar (light blue). (ii) AS07 rotation period ("Fast") and $P_{transition}$ = 1 Mbar (red), 2 Mbar (orange).

helium-rich to helium-poor occurs ($P_{transition}$) is between 1 and 4 Mbars. The heavy elements are assumed to be homogeneously mixed within the planetary envelope. In order to account for the uncertainty linked to differential rotation, we set the uncertainty in the gravitational harmonics to be δJ_2 = -80×10^{-6}, δJ_4 = +20×10^{-6}, δJ_6 = +10×10^{-6} (Hubbard, 1982). Cassini's Js are set to be $J_2 = 16251 \pm 40 \times 10^{-6}$, $J_4 = -926 \pm 11 \times 10^{-6}$, and $J_6 = 81 \pm 11 \times 10^{-6}$. The model gravitational moments are calculated for the 60,330 km reference radius. More details on the interior model can be found in Guillot (2005) and references therein. Finally, we consider two SB rotation periods: the Voyager radio period (labelled "slow"), and the AS07 rotation period (labelled "fast").

Fig. 2 shows the interior model solutions for the two rotation periods. For the 'slow" case, a large variety of solutions is found. The core mass and amount of heavy elements in the envelope are both found to depend crucially on the helium transition level. For 1 Mbar, a relatively large core (10 - 20 $M_\oplus$) and a small amount of heavy elements in the envelope (< 5 $M_\oplus$) are required. For larger values of $P_{transition}$, the core mass decreases while the heavy element masses in the envelope increases. Once $P_{transition}$ =4 Mbar, only a small set of solutions is found. While no solutions with core masses smaller than 4 $M_\oplus$ were found, we believe that this is due to the numerical algorithm used and that *solutions with no core are possible*, as in the case of Jupiter. The ensemble of solutions is more limited for the "fast" rotation period. First, solutions are found only for transition pressures of 1 and 2 Mbar. Second, the solutions span a range of heavy element masses in the envelope that is typically about half of the values found for the "slow" case. The gray areas in Fig. 2 represents a "forbidden zone" with abundances that are lower than the spectroscopic determination. All model solutions for the AS07 rotation period are in that zone and are hence excluded although solutions are likely to be found when a discontinuity in the heavy elements distribution in the envelope is allowed, due to the additional degree of freedom that is then introduced. *Clearly, the choice of the deep rotation period used for the interior models thus has a significant effect on the inferred composition of the planet.*

5. Conclusions

While accurate measurements of the gravitational fields of Jupiter and Saturn can provide tighter constrains on their interior structures the lack of knowledge on the depth of differential rotation is a major source of uncertainty on the internal structures and global composition of the planets. We suggest that radio occultation measurements can be crucial to test the shape models and could lead to constraints on the planets' rotation profiles and departures from hydrostatic equilibrium. Finally, we suggest that in order to make full use of *Cassini* and *Juno* data, non-hydrostatic contributions and various rotation profiles should be incorporated self-consistently in interior structure models.

References

[1] Anderson, J. D. and Schubert, G., 2007. Science, 317, 1384.

[2] Guillot, T., 2005. Annual Review of Earth and Planetary Sciences, 33, 493.

[3] Helled, R. and Guillot, T., 2013. ApJ, 767, 113.

[4] Hubbard, W. B., 1982. Icarus, 52, 509.

[5] Lindal, G. F., Sweetnam, D. N., and Eshleman, V. R., 1985. ApJ, 90, 1136.

[6] Read, P. L., Dowling, T. E. and Schubert, G., 2009. Nature, 7255, 608.