EPSC Abstracts
Vol. 8, EPSC2013-300, 2013
European Planetary Science Congress 2013

The effect of resonance order decrease in N+2 body problem

A. E. Rosaev
OAO NPC NEDRA, Yaroslavl, Russia (hegem@mail.ru)

Abstract

In the restricted three body problem, test particle has strong perturbation at close encounter with the planet during each k-th revolution. For case N+2 body problem strong perturbed encounters may take place in N-times more frequently for the special values of N. In fact, we have the lower order resonance in N+2 than in 3-body problem, i.e. 2:1 instead of 10:9. There are more significant variations in the orbital elements expected in this case. Moreover, the measure of strong perturbed orbits is large in N+2 case, it makes capture in resonance more easy. There are few series of calculations for test particle motion in gravity field of the central mass M and N masses m in vertex of the regular polygon was done to prove this conjecture. The main result of our numeric calculation is that perturbations in the orbital elements, and first of all in the longitude, are more significant in resonance than in non-resonance values of N for the fixed total mass of ring mN as well as for the fixed mass of particle m. The potential of number of continues astrophysical disks and rings can be approximate by the regular N-gon potential. It is necessary to take into account the decreasing of the order of resonance at the various astrophysical disks modeling.

1. Introduction

A system of N points each has mass m, in vertex of the regular N-gon and central mass M is considered. The planar motion of infinitesimal particle close to outer (10:9) and inner (12:13) resonances is studied. The geometry of the problem is given in Fig.1. The discrete and continues model of ring or a disk is used for different astrophysical applications [1,2]. *In the restricted three body problem (RTBP), each mth of it's period, test particle has strong perturbation – close encounter with a planet. For case N+2 body problem strong perturbed encounters take place in N-times more frequently. There are more significant variations in orbital elements expected in this case.*

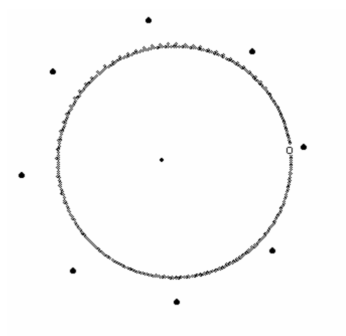

Figure 1: The problem geometry.

First, we consider 3-body problem. At conditions:

$$(a/R)^{3/2} = p/P = n:m > 1 \quad outer \tag{1}$$

$$(a/R)^{3/2} = p/P = n:m < 1 \quad inner \tag{2}$$

where *a, R* and *p, P* – semimajor axis and periods of test particle and particle of ring respectively, resonance effects in motion test particle take place. Most interesting resonance cases are:

$$p/P = (n-2)/n \tag{3}$$

Conjunction. At fixed total mass of N-gon, $m=m_i N$, interaction test particle with regular N-gon strongly depends on number of N.

$$p/P = k(n+2)/(nN), \quad k = 1...N \tag{4}$$

In fact, we have the lower order resonance in N+2 than in 3-body problem, i.e. 2:1 instead of 10:9. At limit N→ ∞ there are no any resonant phenomena. At small N and fixed total mass of ring we have chaotic motion due to close encounters. If N sufficiently large, effect of initial phase not significant. First of all, the possible applications are the protoplanetary disks, but for any clumped disk and rings it may be significant.

2. Numeric modelling

Let us to consider two orbits with initial semimajor axis a=1.072765R (respect to 10:9 resonance) and a= 0.95R. There are two series of calculations: at fixed mass of ring particle and at fixed total mass of ring Nm=0.0001M at different N. The Runge – Kutta integrator is used with step 87.658 seconds. Central mass M=1.989E+30 kg, central distance of each mass m R=1.49E+11 meters. Orbits of all bodies are in same fixed plane. Main results are given in fig.2 -3.

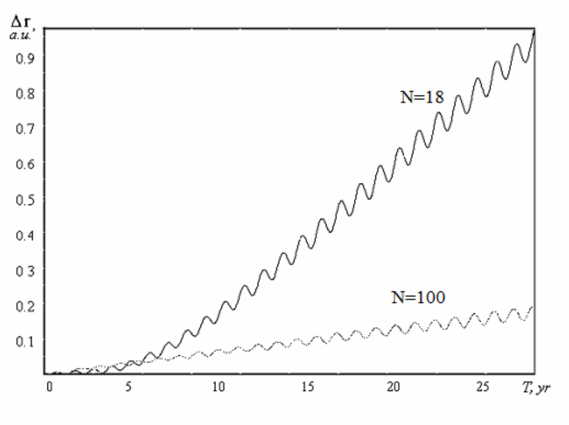

Figure 2: The difference between perturbed and unperturbed positions for resonance case N=18 and for case N=100, close to continuous limit.

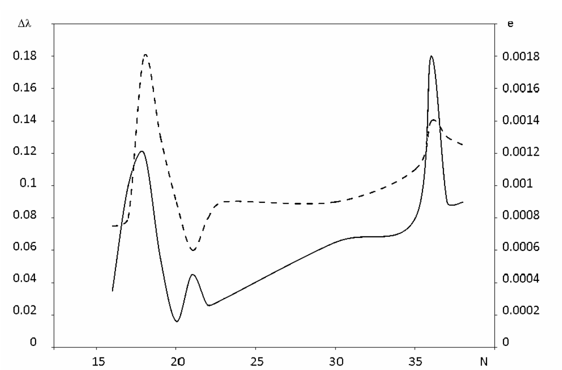

Figure 3: The longitude (solid line) and eccentricity (dashed line) on number of ring particles N dependence.

Semimajor axis of text particle is a=1.073. Fixed mass of ring particle m=10^{-6}M. Fig 2 shows, that test particle shifts relative non-perturbed keplerian ellipse more significant in case discrete ring with resonance N than in quasi-continue case. As it seems in fig.3, this effect takes place not only for case fixed mass of ring, but for fixed mass ring particle too.

3. Summary and Conclusions

A system of N points each has mass m, in vertex of the regular N-gon and central mass M is considered. The planar motion of infinitesimal particle close to outer (10:9) and inner (12:13) resonances is studied. In the restricted three body problem, test particle has strong perturbation at close encounter with the planet during each k-th revolution. For case N+2 body problem, we have the lower order resonance in N+2 than in 3-body problem, i.e. 2:1 instead of 10:9. There are few series of calculations for test particle motion in gravity field of the central mass M and N masses m in vertex of the regular polygon prove this conjecture. There are more significant variations in the orbital elements detected in this case. The main result of our numeric calculation is that perturbations in the orbital elements, and first of all in the longitude, are more significant in resonance than in non-resonance values of N for the fixed total mass of ring mN as well as for the fixed mass of particle m. Moreover, the measure of strong perturbed orbits is large in N+2 case, it makes capture in resonance more easy. The potential of number of continues astrophysical disks and rings can be approximate by the regular N-gon potential. This order of resonance decreasing is necessary to take into account at the various astrophysical disks modelling.

References

[1] Alberti A., Vidal C. : Dynamics of a particle in a gravitational field of a homogeneous annulus disk, Celestial Mech Dyn Astr, Vol. 98, pp.75-93, 2007.

[2] Hure J-M., Hersant F.: The Newtonian potential of thin disks, arXiv 1104.5079v1 [astro-ph.IM], 27 Apr 2011