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Corrections of the critical parameters of almost adiabatic convection in liquid cores of terrestrial planets

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Abstract

The convection heat transfer resulting from the flow between two rapidly rotating concentric spheres in almost adiabatic approximation is considered. For different but sufficiently thick fluid shells we obtain fully analytical expressions for critical Rayleigh numbers, frequencies and distributions of planetary/moons convection for all possible Prandtl numbers in the approximation of small inner to outer radius aspect ratio with the first order corrections. We consider non-uniform distribution of convective sources with the maximum source power at the inner boundary which forms when the heat transfer rate differs on the inner and outer boundaries and power of heat sources in the volume of the spherical layer is negligibly small. Similar distribution of thermal and/or compositional convective sources is likely typical for the deep convective interiors of the most part of the planets and moons since it is possible to neglect a homogeneous radioactive heating.

1. Introduction

The present work forms the continuation of the paper [6], in which the basic equations describing marginal stability of almost adiabatic thermal and compositional convection in the deep planetary interiors was obtained and partially investigated. All considered objects, planets and moons of Terrestrial type, are in the state of rapid rotation, which means that fluid motion in its deep convective interiors is characterized by the large Reynolds number, or, equivalently, by the small Ekman number. The presence of this small parameter allows us using the asymptotic analysis to reduce the initial problem for the system of partial differential equations to the simplified two-point boundary value problem for a single second-order ordinary differential equation. In this paper we consider sufficiently thick spherical layers. The latter means that the inner radius r_i of the spherical layer is much smaller than the outer radius r_o. Then it is possible to consider their ratio $b = r_i/r_o$ as an additional small parameter in the system which allows us to use WKBJ approach and find analytical solution for the system of two ODEs for the vertical velocity and the pressure. In this case we also assume that the heat source distribution is non-uniform with maximum of heat power on the inner boundary. For example, this distribution corresponds to the situation when the temperature gradient is maintained by the temperature difference between the inner and outer boundaries. Or, more naturally, when the heat transfers from the inner and outer boundaries are different and the power of the heat sources in the spherical volume is negligibly small.

2. General solution for small inner core

Following [1,2,3] we construct the leading order of WKB-type solution and corresponding dispersion relation. We will aim to keep in the equations maximum number of terms in the leading order of series in asymptotically small parameter b and using the conditions (here R is the Raleigh number and m is the azimuthal number):

$$\mathrm{Im}(R) = 0, \quad \mathrm{Im}\left(\frac{\partial R}{\partial m}\right) = 0, \qquad \text{(23a,b)}$$

we receive following relations for the Raleigh number, frequency and the azimuthal number of the almost adiabatic convection in the leading order:

$$m(b,\mathrm{Pr}) = b^{2/3}\frac{\mathrm{Pr}^{1/3}}{2^{1/6}}(1+\mathrm{Pr})^{1/3},$$

$$\omega(b,\mathrm{Pr}) = b^{-2/3}\frac{2^{1/6}}{\mathrm{Pr}^{1/3}(1+\mathrm{Pr})^{2/3}}, \qquad (25a,b)$$

$$R(b,\mathrm{Pr}) = b^{-1/3}\frac{3\,\mathrm{Pr}^{4/3}}{2^{2/3}(1+\mathrm{Pr})^{4/3}}. \qquad (25c)$$

Comparison with our numerical results and with numerical results [2] for the fixed values b=0.35 and Pr=1 shows a qualitative agreement if we take into account that the sign of frequency value is not important. To receive better agreement we also calculate first-order corrections to our leading-order solution.

6. Summary and Conclusions

To find the critical Rayleigh numbers, frequencies and distributions of almost adiabatic planetary convection for all possible Prandtl numbers in the approximation of small radius aspect ratio b we consider non-uniform distribution of convective sources with the maximum source power at the inner boundary which forms when the heat transfer rate differs on the inner and outer boundaries and power of heat sources in the volume of the spherical layer is negligibly small [5]. Similar distribution of thermal and/or compositional convective sources is likely typical for the deep convective interior of the most part of the planets and moons since it is possible to omit a homogeneous radioactive heating [4].

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