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Transport coefficient free scaling laws for convection and magnetism in fast rotating planets

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Abstract

In the limit of negligible molecular diffusivity, viscosity and magnetic diffusivity effects, Scaling laws for convection and magnetism are derived for fast rotating planets. In the Earth, Jupiter, Saturn and ancient dynamo active Mars it is reasonable to suppose domination of magnetic energy over kinetic one that results in the typical magnetic field B proportional to the third root of the buoyancy flux F [3] driving the convection, while B is independent on conductivity σ and angular rotation rate Ω. The same scaling law was previously obtained via compilation of many numerical planetary dynamo simulations [1-3]. Besides, new scaling laws are obtained for typical hydrodynamic scale h, velocity V, Archimedean acceleration A, electromagnetic scale d and sinus of the angle between magnetic and velocity vector s. In Uranus, Neptune and Ganymede a local magnetic Reynolds number $r_m=\mu\sigma Vd\sim 1$ with magnetic permeability in vacuum μ. Correspondent magnetic energy could be of order kinetic energy resulting in relatively lower magnetic field strength $B=(\mu\rho)^{1/2}V$ with density ρ. That may explain magnetic field values and non-dipolar structures in Uranus, Neptune and Ganymede.

1. Convection scaling

Following [2, 6, 7] heat-mass-transfer effects give

$$AV = F\ . \qquad (1)$$

Radial component of curl of momentum equation (e.g. eq. (3) in [6]) contain no radial buoyancy force $\sim A$. Due to non-penetrating conditions on velocity I also remove the Coriolis force from this component integrating it along the axis of rotation from one spherical boundary of the dynamo region to another. That equating magnetic to kinetic energy ratio to $(d/h)^2$ balances magnetism and inertia anywhere as

$$B^2/d^2\mu\rho = V^2/h^2\ . \qquad (2)$$

Using (2) and other components of the same curl give

$$\Omega V/H = V^2/h^2 = A/h \qquad (3,\ 4)$$

in conditions of fast rotation. H is thickness of the spherical shell where the dynamo is acting. Solving equations (1, 3, 4) I obtain Rhines [4, 7] scaling laws for convection in all the fast rotating planets:

$$h = (FH^3/\Omega^3)^{1/5}, \qquad A = (F^3\Omega/H)^{1/5}, \qquad V = (F^2H/\Omega)^{1/5}. \qquad (5,\ 6,\ 7)$$

2. Electromagnetic scaling

Faraday's law with typical electric field E on large dipolar scale and Ohm's law with $r_m \gg 1$ give us

$$VB/H = E/d\ ,\ E = sVB\ . \qquad (8,\ 9)$$

Neglecting by magnetic diffusivity terms and using $d \gg h$ in the induction equation (e.g. eq. (2) in [6]) I estimate the inverse time of magnetic field change as

$$sV/h = Vd/Hh\ . \qquad (10)$$

Supposing that the work of Archimedean force is of the order of magnetic energy time-change I estimate

$$B^2Vd/Hh\mu = \rho AV\ . \qquad (11)$$

This and equations (2, 5-7) give the first principles' scaling law known previously only from compilation (e.g. [1-3]) of many numerical simulations:

$$B = (\mu\rho)^{1/2}(FH)^{1/3}. \qquad (12)$$

Finally, for r_m>>1 and corresponding domination of magnetic energy over kinetic one [7] or $d>>h$ from (2), the remained scaling laws for typical values are

$$d = \frac{Bh}{\sqrt{\mu\rho}\, V}, \quad s = \frac{d}{H}, \quad E = \frac{VBd}{H}. \qquad (13, 14, 15)$$

For moderate or small local magnetic Reynolds number r_m magnetic energy could be of order kinetic one or $d=h$ in according with (2) and I estimate

$$B = (\mu\rho)^{1/2} V \; . \qquad (16)$$

3. Planetary MHD dynamo values

Table 1: Typical values where kinetic energy is of order magnetic one and for moderate or small r_m

Value, unit	Ganymede	Uranus	Neptune
F, 10^{-14} m^2/s^3	6	3000	5000
Ω, 10^{-5}/s	10	10	11
ρ, Mg/m^3	8	2.5	3
H, Mm	0.5	4	5
B, µT	15	500	700
$h=d$, km	1	20	25
V, mm/s	0.2	8	12
$r_m=\mu\sigma Vd$	0.2	1	1.5

In Table 1 I use idea of [5] that dynamo in Uranus is operating in slightly thinner layer than in heavier Neptune. Besides, following [6] I estimate buoyancy flux F on the base of 70% from the maximal possible heat fluxes observed [1] on the surface of Uranus and Neptune. Input values (first 4 rows) are from [3], while in contrast to [1-3] to be consistent with the observable magnetic fields equations (5-7, 16) are used. Planets with magnetic energy exceeding kinetic one and large r_m >>1 are presented in Table 2. Input values (first 4 rows) are also from [3]. Similar to [1-3] scaling laws described by equations (5-7, 12) are used to be consistent with the observable and estimated (for ancient Mars) magnetic fields. On top of that I use equations (13, 14) to utilize new scaling laws. Those may be useful for estimation of turbulent transport coefficients (via V, h and d), magnetic to kinetic energy ratio = $(d/h)^2$ and for relative geometry of magnetic and velocity field via s.

Table 2: Typical values for planets with magnetic energy exceeding kinetic one and large r_m >>1

Value, unit	Mars	Earth	Jupiter	Saturn
F, 10^{-13} m^2/s^3	4	2	200	100
Ω, 10^{-5}/s	7.3	7.3	17.7	16.4
ρ, Mg/m^3	10	11	1.8	1.8
H, Mm	1.1	2.3	41	16
B, mT	1	2	5	3
h, km	4	6	45	25
d, km	30	90	500	250
V, mm/s	1.2	1	9	6
$s=d/H$	0.03	0.04	0.01	0.02
$r_m=\mu\sigma Vd$	100	150	350	120

I intentionally did not consider in this work Mercury and Venus because they are rotating not so fast to be consistent with the presented estimations.

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