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# Markov chain Monte-Carlo orbit computation for binary asteroids

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## Abstract

We present a novel orbit comutation method for resolved binary asteroids. The method relays on Markov chain Monte Carlo sampling of orbital period and three observations, the parameters used in the Thiele-Innes method. We threat the orbit computation problem as an inverse problem and apply Baysien statistical methods to solve for the maximum likelihood orbit and confidence regions.

## 1 Orbital inverse problem for binary asteroids

In the orbital inverse problem for binary asteroids the orbital parameters of the apparent orbit are solved for given the observations, that is the cartesian positions of the secondary component of the system with respect to the primary. The below treatment follows from the theory of asteroid orbital inverse problem for single asteroids [Virtanen et al.(2001), Muinonen and Bowell(1993), Bowell et al.(2002), Virtanen et al.(2005), Oszkiewicz et al.(2005)]. The solution to the binary orbital inverse problem can be written similarly as for single asteroids as an a posteriori probability, using Bayes's theorem:

$$p_p(\mathbf{P}) = Cp_{pr}(\mathbf{P})p(\psi \mid \mathbf{P}) \quad = \quad Cp_{pr}(\mathbf{P})p_\epsilon(\Delta\Psi(\mathbf{P})),$$

where: $\mathbf{P}$ denotes the unknown osculating elements $(a, e, i, \Omega, \omega, P, \mathcal{M})^T$ at epoch $t_0$; $\psi$ denotes a set of given astrometric observations, consisting of cartesian positions of the secondary component that is $x, y$ pairs: $\psi = (x_1, y_1; ...; x_N, y_N)^T$ at N different dates; $p_{pr}(\mathbf{P})$ is the a priori probability density function (p.d.f.), which can be selected to be informative and contain for example some expert knowledge on the values of the orbital parameters $\mathbf{P}$; $p_\epsilon(\Delta\Psi(\mathbf{P}))$ is the observational error p.d.f. (usually being assumed Gaussian), evaluated for the observed-minus-computed (O-C) residuals.

To characterize the complicated a posteriori target densities, we use Monte-Carlo methods. In particular, we focus on Markov-Chain Monte-Carlo technique combined with the Thiele-Innes method.

## 2 Thiele-Innes method

Thiele-Innes method is an orbit computation method for binary asteroids based on three observations and orbital period (or equivalently the areal constant) [Hestroffer et al.(2010)]. The method was developed in the $19^{th}$ century [Aitken(1964)] and is still a popular orbit computation for resolved binary stars and asteroids. The method directly provides (if it exists) a Keplerian elliptic orbit, that can be expressed for example in terms of keplerian elements $\mathbf{P} = (a, e, i, \Omega, \omega, P, \mathcal{M} = (m_1 + m_2))$, which are, respectively, the semimajor axis, eccentricity, inclination, longitude of ascending node, argument of perihelion, orbital period, and mass of the system. The method requires a minimum of 3 observational data points $\psi_i = (x_i, y_i)$ in the same tangent plane (where $x, y$ are the cartesian positions of the secondary component of the binary system with respect to the primary component) at times $t_i$ for $i = 1, 2, 3$ and orbital period $P$ or the areal constant $c$. The orbit computation is based on the fundamental equation relating the eccentric anomaly $E$ and the double areal constant $c$ to the orbital period $P$ [Hestroffer et al.(2010)]:

$$\begin{aligned} t_k - t_l - \Delta_{lk}/c &= n^{-1}[E_k - E_l - \sin(E_k - E_l)] \\ \Delta_{pk} &= x_l y_k - x_k y_l \\ n &= 2\pi/P \end{aligned}$$

where $(l, k) \in [1, 2, 3] \times [1, 2, 3]$ and $l \neq k$. The transcendental set of equations can be itteratively solved for the areal constant $c$ and consequently the orbital elements can be computed.

# 3 Markov chain Monte-Carlo sampling

To sample the possible orbital solutions we make use of the Metropolis-Hastings (M-H) algorithm. First, from the whole set of $N$ observations $\psi_i = (x_i, y_i)$ made at observation times $t_i$ (where $i = 1..,N$) we randomly select three observations from the same tangent plane and a starting orbital period $P$. We refer to those seven parameters as the sampling parameters and denote by $\mathbf{S} = (x_1, x_2, x_3, y_1, y_2, y_3, P)$. From the three selected observations and the period we compute a starting orbital elements $\mathbf{P} = (a, e, i, \Omega, \omega, P, \mathcal{M})$ using the Thiele-Innes method. Once a starting orbit have been computed we start Markov chain Monte-Carlo (MCMC) sampling of the parameters $\mathbf{S}$ by adding random deviates to the selected three observations and the orbital period.

In practice at each iteration $t$ in a chain a new candidate sampling parameters are proposed using so-called proposal densities. In particular we make use of Gaussian proposal densities for all the seven sampling parameters. For the cartesian coordinates we use proposal densities that are centered around the last accepted sampling parameters in the chain and the size of the proposal density is proportional to the observational noise: $x_i^{(c)} \propto N(x_i^{(t-1)}, \sigma_{x_i})$, $y_i^{(c)} \propto N(y_i^{(t-1)}, \sigma_{y_i})$ (where $i = 1, 2, 3$) for $x$ and $y$ coordinates respectively. For the orbital period we use a normal distribution that is centered around the last accepted period $P^{(c)} \propto N(P^{(t-1)}, \sigma_P)$. The size of the proposal density for the orbital period $\sigma_P$ is the only parameter to be tuned in the method, but in general in most of the cases, an educated guess of the size of that parameter can be made. If the correlations between the different observations are known, they could also be utilized as in the proposal density.

Once a new candidate sampling parameters have been generated
$\mathbf{S}^{(c)} = (x_1^{(c)}, x_2^{(c)}, x_3^{(c)}, y_1^{(c)}, y_2^{(c)}, y_3^{(c)}, P^{(c)})$ the acceptance coefficient $a_r$ is computed as:

$$a_r = \frac{p_p(\mathbf{P}^c)}{p_p(\mathbf{P}^{t-1})} \frac{\mid J^{t-1} \mid}{\mid J^c \mid},$$

where $J^{(c)}$ and $J^{(t-1)}$ are the Jacobians from the sampling parameters to orbital parameters for the candidate and the last accepted orbit respectively. $\mathbf{P}^{(c)}$ and $\mathbf{P}^{(t-1)}$ are the p.d.f.s. for the candidate and the last accepted orbit respectively. Next the candidate parameters are accepted or rejected based on Metropolis-Hastings criteria:

$$\begin{array}{ll} \text{If } \mathrm{a_r} \geq 1, & \text{then } \mathbf{P}_\mathrm{t} = \mathbf{P}^\mathrm{c}. \\ \text{If } \mathrm{a_r} < 1, & \text{then } \left\{ \begin{array}{l} \mathbf{P}_t = \mathbf{P}^c, \text{ with prob. } \mathrm{a_r}, \\ \mathbf{P}_t = \mathbf{P}_{t-1}, \text{ with prob. } 1 - \mathrm{a_r}. \end{array} \right. \end{array}$$

In practice if the new orbit produces a better fit to the full observational data set, it is always accepted. If it produces a worse fit, it is accepted with the probability equal to $a_r$. The sampling is repeated until a large enough number of orbits have been obtained. After the sampling is complited convergence diagnostics has to be performed to insure that the stationary distribution was reached and to test for the lenght of burn-in period (the time required for the chain to reach the stationary).

The obtained distributions of the orbital parameters reflect the properties of orbital element uncertainties.

# 4 Conclusions

We have developed a new orbit computation method for binary asteroids. The method has been implemented into the Gaia mission data processing pipeline.

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