EPSC Abstracts
Vol. 8, EPSC2013-577, 2013
European Planetary Science Congress 2013

Loss of Hyperbolicity for Impact into Geomaterials

D. de Niem
DLR Institute of Planetary Research, Rutherford Str.2, D-12489 Berlin, Germany (detlef.deniem@dlr.de)

Geologists widely use a scale of shock metamorphic phenomena in minerals (Hörz, 1968) and meteorites (Müller and Hornemann, 1969) calibrated under laboratory conditions to map the amount of shock deformation in impact and explosion craters (Stöffler and Langenhorst, 1994; Grieve et al., 1996). Remarkably, low-dimensional planar defects occur in quartz below the pressure required to form stishovite under static conditions (Hörz, 1968; Müller and Hornemann, 1969). An empirical classification scheme exists that extends from the onset of low-dimensional crystallographic defects to a pressure where shock melting produces glass (Stöffler and Langenhorst, 1994; Grieve et al., 1996).

Here it is proposed that loss of hyperbolicity indicates a macroscopic cause behind the formation of some of the microscopic defects. This follows from a characteristic analysis of the differential equations of elastic-plastic flow, augmented by the equation of state (EOS), a plastic flow rule and the Jaumann rate as constitutive model. For a Drucker-Prager-like yield criterion (Willam, 2002), loss of hyperbolicity begins above certain threshold pressure. The criterion of wave instability may be used as additional damage indicator, where conventional numerical methods (Wilkins, 1964; Predebon et al., 1991; Ivanov, de Niem, Neukum , 1997) are insensitive. Let $n_i = (\cos\phi, \sin\phi, 0)$ denote the direction of wave propagation in 2D cartesian or cylindrical coordinates, $t_i = (-\sin\phi, \cos\phi, 0)$ the tangential direction and $N := n_i n_k S^{ik}$, $M := t_i t_k S^{ik}$ and $T := n_i t_k S^{ik}$ are the non-trivial components of the stress deviator S^{ik}. Furthermore $\tilde{N}$ and χ are the following abbreviations

$$\tilde{N} := N + Y \frac{K}{G}\frac{dY}{dp}, \tag{1}$$

$$\chi := 1 + \frac{K}{G}\left(\frac{dY}{dp}\right)^2, \tag{2}$$

where K and G are the compressive and shear moduli, respectively, and a pressure-dependent yield strength $Y(p)$ has been assumed where that of Lundborg (1968) is an example. Then the criterion takes the form

$$\Delta := \frac{GT\tilde{N}}{\rho^2 J_2\chi}\left(T + \frac{GT\tilde{N}}{J_2\chi}\right) \geq c^2_{P,(0)} c^2_{S,(0)}. \tag{3}$$

The generalized P and S wave speeds are given by $c^2_{P,S} = (c^2_{P,(0)} + c^2_{S,(0)})/2 \pm \sqrt{(c^2_{P,(0)} - c^2_{S,(0)})^2/4 + \Delta}$ in terms of

$$\rho c^2_{P,(0)} := K + G\left(\frac{4}{3} - \frac{\tilde{N}^2}{J_2\chi}\right) \tag{4}$$

$$\rho c^2_{S,(0)} := G\left(1 - \frac{T^2}{J_2\chi}\right) + \frac{N-M}{2}. \tag{5}$$

If the inequality in Eq.(3) is valid, P waves remain propagating, only the S wave speed becomes imaginary. The onset occurs for sufficiently high pressure. The criterion depends on few parameters only: the elastic moduli, the Hugoniot elastic limit, and the slope of the yield curve dY/dp also known as the coefficient of friction. Cohesion, i.e. the zero pressure value of Y is less important since it is very low in comparison to the Hugoniot elastic limit for most geomaterials. It is additionally required that a sufficiently large non-diagonal component T is present, unlike the stress state in a plane shock with only a normal particle velocity. Instability requires a wave with a nonzero transverse velocity component. Such waves may form when the main shock is reflected from container walls or inhomogeneities in shock reverberation experiments, such as those by (Müller and Hornemann, 1969; Stöffler and Langenhorst, 1994; Fritz et al. , 2011). The post-shock stress state largely depends on the amount of transversality $|v_y/v_x|$ and this may lead to the considerable range of values reported by (Müller and Hornemann, 1969; Stöffler and Langenhorst, 1994; Fritz et al. , 2011).

At the threshold pressure only a single point on the yield surface becomes unstable, see Fig.1. With increasing pressure the unstable region widens up. The yield surface displayed in Fig.1 is parametrized in terms of two angles α, β defined

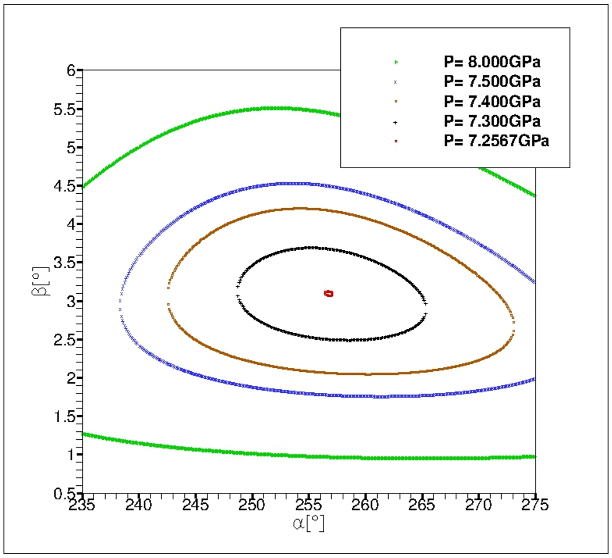

Figure 1: Gradual opening of unstable region for quartz. Yield surface parametrized in terms of two angles α, β: $N + M =: 2/\sqrt{3}Y \sin\beta \cos\alpha$, $N - M =: 2Y \sin\beta \sin\alpha$, $T =: Y \cos\beta$. For given values of P, part inside contours corresponds to unstable state.

with the help of $N + M =: 2/\sqrt{3}Y \sin\beta \cos\alpha$, $N - M =: 2Y \sin\beta \sin\alpha$, $T =: Y \cos\beta$. Using elastic moduli G, K and the other parameters typical for rock materials the range for the onset of the instability can be investigated. For quartz it occurs at about 7.257 GPa, see Fig.1, assuming the following parameters: G=30 GPa, K=37.88 GPa, Y_{HEL}=2.66 GPa, dY/dp=0.85.

Acknowledgements

The author was supported by the Helmholtz Association through the research alliance “Planetary Evolution and Life“.

References

Hörz, F., 1968. Statistical measurements of deformation structures and refractive indices in experimentally shock loaded quartz. In: Shock Metamorphism of Natural Materials (eds. B. French and N. M. Short, Mono Book Corp., Baltimore, Maryland, USA.), 243-253.

Müller, W. F., Hornemann, U., 1969. Shock-induced planar deformation structures in experimentally shock-loaded olivines and in olivines from chondritic meteorites. *Earth Planet. Sci. Lett* **7**, 251-264.

Stöffler, D. Langenhorst, F., 1994. Shock metamorphism of quartz in nature and experiment: I. Basic observation and theory. *Meteoritics* **29**, 155-181.

Fritz, J., Wünnemann, K., Reimold, W. U., Meyer, C., Hornemann, U, 2011. Shock experiments on quartz targets pre-cooled to 77 K. *Intl. J. Impact Engng.* **38**, 440-445.

Grieve, R. A. F., Langenhorst, F., and Stöffler, D., 1996. Shock metamorphism of quartz in nature and experiment: II. Significance in geoscience. *Meteor. Planet. Sci.* **31**, 6-35.

Lundborg, N., 1968. Strength of rock-like materials. *Int. J. Rock Mech Min. Sci.* **5**, 427-454.

Willam, K., J., 2002. Constituitive models for engineering materials. Encyclopedia of Physical Science & Technology, vol. 3., Academic Press, 603-633.

Wilkins, M. L., 1964. Calculation of Elastic-Plastic Flow. In: Methods in Computational Physics vol. 3, Fundamental Methods in Hydrodynamics, Academic Press, New York, 211-263.

W. W. Predebon, C. E. Anderson, J. D. Walker, 1991. Inclusion of evolutionary damage measures in Eulerian wavecodes. *Comput. Mech.* **7**, 221-236.

Amsden, A.A., H.M.Ruppel, and C.W.Hirt. 1980. SALE: A Simplified ALE Computer Program for Fluid Flow at All Speeds. Los Alamos National Laboratory Report LA-8095, Los Alamos, NM, 101 pp.

Ivanov, B. A., de Niem, D., and Neukum, G., 1997. Implementation of dynamic strength models into 2D hydrocodes: Applications for atmospheric breakup and impact cratering *Intl. J. Impact Engng.* **20**, 411-430.