

Origin of proto-planetary rings in the Solar system

T. Abdulmyanov

Kazan State Power Engineering University, Kazan, Russia (abdulmyanov.tagir@yandex.ru / Fax: +7-843-5624356)

Abstract

This paper presents a model of the surface wave disturbances of proto-planetary disk of the Solar system. With the help of an analytic solution of the wave equation for a circular disk the locations of proto-planetary rings are determined. The mechanism of particle migration from the outer zones in the proto-planetary rings is considered.

1. Introduction

Formation of the solar system, according to recent research, could start with the fragmentation of the proto-planetary disk and the formation of proto-planetary rings. Fragmentation of the proto-planetary disk can appeared as results of wave actions [1]. Circular waves can occur as a consequence of active star formation processes in the early stages of the proto-planetary disk. Such waves are observed only in supernova explosions of stars. However, around other stars, such waves can exist, but can be optically inaccessible. Circular waves around other stars could be due to the large density gradient in the central part of the disc and its periphery. In this case, the surface of the proto-planetary disk that is in fluid balance, there will always be a good conductor of any central disturbances, even of small amplitude.

2. Origin of the surface wave disturbance of proto-planetary disk

Compression (collapse) dynamics of the proto-stellar cloud was first investigated in 1964 by the Japanese astrophysicists C. Hayashi and T. Nakano. The compression process proto-star clouds can be divided into four main phases: the stage of rapid compression proto-stellar clouds, the FU Orionis stage, the T Tauri stage, the stage of thermonuclear processes in the Sun. (i) At the stage of rapid compression proto-stellar clouds, which lasts for about 10 years [3], the radius of the proto-star down from 40 AU to the value of semi-major axis of the orbit of Mercury (about 0,4 AU). That is, the radius of the proto-star

dropped a hundred times. Spatial density of the proto-star in this case will increase a million times. This change in density in a relatively short time (about 10 years) will lead to the replacement of the light gases, and emissions from the central proto-star to the periphery and outside of the proto-star. The movement of these gases will be created on the surface of the forming of the proto-planetary disk surface waves. (ii) At the FU Orionis stage (from 10^2 years to 10^5 years from the beginning of the rapid compression proto-star), the radius of the proto-planetary disk was about 5 AU. By the end of this stage, the radius of the proto-star is decreased by 10 times and was about 0,04 AU. The density was increased by almost a thousand times. (iii) The T Tauri stage is the stage of slow compression proto-star and the longest stage (from 10^5 years to $4 \cdot 10^6$ years from). By the end of the stage of T Tauri proto-star radius is decreased by 10 times and came close to the current radius of the Sun (about 0,00465 AU). The spatial density of the proto-star has increased a thousand times. The value of the radius of the proto-planetary disk at that time was around 0,18 AU. (iiii) The stage of thermonuclear processes in the Sun (from $4 \cdot 10^6$ to $5 \cdot 10^6$ years from the beginning of the rapid compression proto-star).

Thus, the process of compression proto-star there is a big difference between the pressure inside the contracting proto-star in the emerging field of the proto-planetary disk. The difference in density of the proto-star and the surface of the proto-planetary disk will be created on the surface of the proto-planetary disk regular periodic disturbance. These disturbances can not be redeemed due to the stability of Kepler disk. Such perturbations of the surface of the proto-planetary disk are the most interesting in the study of the mechanism of formation of proto-planetary rings.

3. Formation of the rings

The most intensive movement of the waves could be even in the early stages of the proto-planetary disk. Wave motion of gas and dust particles are formed mainly by the action of two forces [2]: by the gravitational force $\delta F_g = 4\pi\mu\rho_0 u$, related to a change

$\delta\rho$ density $\rho = \rho_0 + \delta\rho$ and by the gas pressure $\delta F_P = \rho(\partial P/\partial x + \partial P/\partial y) \approx c_s^2 \Delta u$, where $u(x, y)$ - the displacement of gas and dust particles as a result of the forces δF_g and δF_P , c_s - the speed of sound, $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$. The condition of gravitational instability in this case will be the following: $c_s^2 \Delta u < 4\pi\mu\rho_0 u$, where μ - is the shear modulus. Then the acceleration of particles in the polar coordinates (r, φ) can be determined from the following equation: $u_{tt} = c_s^2 \Delta u + 4\pi\mu\rho_0 u$. To solve the equation will apply the Fourier method. As a result we obtain the function $u(r, \varphi, t)$:

$$u(r, \varphi, t) = \sum_{v=0}^{\infty} \Phi_v(\varphi) \sum_{k=0}^{\infty} T_{kv}(t) R_{kv}(r),$$

where $\Phi_v(\varphi) = \cos(v\varphi + \varphi_0)$, $R_{kv}(r) = J_v(\lambda_{kv}r/R_0)$, J_v - the Bessel functions of order v , λ_{kv} - zeros of the Bessel functions J_v ; R_0 - radius of the proto-planetary disk, φ_0 - the arbitrary constant, a_{kv} , b_{kv} - the coefficients determined by the initial conditions for the equation of acceleration. If the value of the constant $\lambda = [4\pi\mu\rho_0 - (\lambda_{kv}/R_0)^2] > 0$, then the function T will be equal $T_{kv}(t) = a_{kv}\exp(c_s\lambda^{1/2}t) + b_{kv}\exp(-c_s\lambda^{1/2}t)$. If the value of the constant $\lambda \leq 0$, then $T_{kv}(t) = a_{kv}\cos[c_s(-\lambda)^{1/2}t] + b_{kv}\sin[c_s(-\lambda)^{1/2}t]$. The initial and boundary conditions are defined as follows: $u(r, \varphi, 0) = f(r, \varphi)$, $u_t(r, \varphi, 0) = 0$, $|u(R_0, \varphi, t)| < 0$, where $f(r, \varphi)$ - the function that defines the shape of the initial surface of the proto-planetary disk. Shape of the surface of the proto-planetary disk is defined using the results obtained in [2]. Depending on the temperature of the dust layer at a distance R from the center of the proto-planetary disk [2], the ratio $\beta = h/R$ will increase the value of 0,019 at the Mercury, to a value of 0,033 at the level of the giant planets, where h - half the width of the proto-planetary disk distance R . The average value of the parameter β is equal to 0,026. Consequently, the growth of the height of the proto-planetary disk, to a first approximation, is a linear function of the polar radius r : $h(r) = 0,026 \cdot r = f(r)$. For this case coefficients a_{kv} can be defined in the following form: $a_{kv} = 2\beta R_0/\lambda_{kv}$, $b_{kv} = 0$. In Fig. 1 shows the form of the function $u(r, \varphi, t)$ for the parameter $R_0 = 5$ AU. According to Fig. 1 when the parameter $R_0 = 5$ AU, for the values $r = 10, 20, 30, 40$ (AU) are formed maxima of the function $u(r, \varphi, t)$. Because of these features waves, dust particles will settle in the intervals between the peaks. The masses of the particles in the gaps will grow faster than near

the maxima. However, these particles will experience greater inhibition by the surrounding gas and the particles will move towards the center of proto-planetary disk. As a result, the dust particles will accumulate near the maxima.

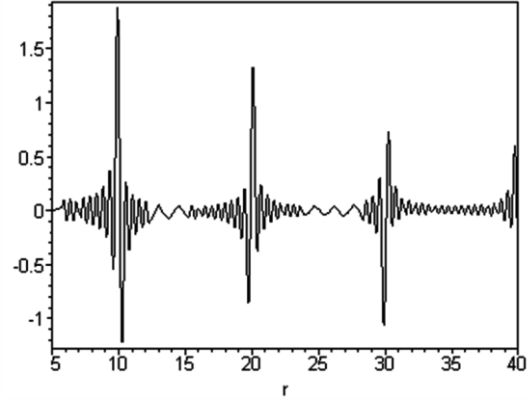


Figure 1: The location of the singularities of waves $u(r, \varphi, t)$ for the parameter $R_0 = 5$ AU, $5 \leq r \leq 40$ (AU), $(\varphi = 0, t = 3)$.

4. Summary and Conclusions

According to the wave model of surface disturbances of the proto-planetary disk, the dust particles will accumulate near the highs of the perturbation, forming a proto-planetary ring. Proto-planetary rings of Pluto, Neptune, Uranus and Saturn were formed at the stage of FU Orionis. Proto-planetary rings of Jupiter and asteroids belt were formed at the stage of T Tauri. Proto-planetary rings of terrestrial planets were formed at the beginning stage of thermonuclear reactions in the Sun.

References

- [1] Abdulmyanov, T. R.: The evolution of the Solar activity and planet formation, EPSC2012-75, Vol. 7, 23 – 28 September, Madrid, Spain, 2012.
- [2] Safronov, V. S.: Evolution of the proto-planetary cloud and formation of the Earth and other planets, English translation NASA TT F-677, 1972.
- [3] Shklovsky, I. S.: Stars of their birth, life and death. Moscow, Nauka, 1977.