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Evolution of proto-planetary rings in the Solar system

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Abstract

This paper presents the mechanism of accumulation of dust particles in the proto-planetary rings. With the help of an analytic solution of the wave equation for a circular disk the condition of the gravitational instability is determined. The critical value of the density of dust particles is also defined.

1. Introduction

Dynamic structure of the asteroid belt shows that the slow growth of bodies in less dense proto-planetary rings could be the cause of the simultaneous formation of many bodies which has heliocentric orbits. In such cases, the mechanism of mutual collisions of bodies is becoming one of the main mechanisms of the orbital evolution of these bodies. Rings with a critical density, by gravitational instability will collapse. According to the current researches the planet formation process can be divided into several stages, depending on the size of formed bodies. The first stage is the formation of small particles (a < cm), the second stage - the formation of meter-sized bodies. The third stage is the formation of bodies from one to ten kilometers in size, the fourth stage - the formation of bodies of more than ten kilometer's size. The main mechanisms of growth planetesimals to the size of the planets could be mechanisms of long-period librations near the Lagrange points [3]. For small planets it will be near the mean motion commensurabilities $(p + q) / p$ [1]. One of the most difficult to modeling is the form and growth of body size of about one meter.

2. The gravitational instabililty condition in the rings

The condition of the gravitational instability $\delta F_g > -\delta F_P$ [4] can be written as follows:

$$c_s^2\,\Delta u < -\,4\pi\mu\rho_0 u,\ \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2. \quad (1)$$

Substituting into (1) the partial solutions $u_{k\nu}(r, \varphi, t) = T_{k\nu}(t)\cdot\Phi_\nu(\varphi)\cdot R_{k\nu}(r)$ of equation $u_{tt} = c_s^2\,\Delta u + 4\pi\mu\rho_0 u$ the gravitational instability condition for circular waves will be the following: $\rho > \rho_0 = (2\pi\mu)/c_s^2$, where c_s – sound speed, $\mu = \rho \cdot c_s^2$ – the shear modulus. The condition of the gravitational instability according to the Jeans test limits sinusoidal perturbations lengths [4]: $\lambda^2 > \lambda_c^2 = \pi \cdot c_s^2/(\mu\rho_0)$. The rings with a critical density, by gravitational instability will collapse. The compression will not be local, if and only if the critical density will support by influx of particles from the outer regions of the ring. If the collapse is not local, then in the ring will be formed single planetesimal. The width and density of proto-planetary rings of the Solar system will determine the outcome of the present values of the masses of the planets, major axis and eccentricity of their orbits, as well as the movement of the light gases from the region of the terrestrial planets in the zone of the giant planets. The width of the proto-planetary ring is determined by the interval $[r_p - r_0, r_a + r_0]$, where $r_p = a(1 - e)$, $r_a = a(1 + e)$, $r_0 = \sqrt[5]{m}$, m - mass of the planet, a, e - semi-major axis and eccentricity of orbit of the planet [2]. Table 1 shows the spatial (ρ) and surface (ρ_1) density of proto-planetary rings calculated by formulas: $\rho = m / V$, $\rho_1 = m / S$, $\rho_2 = 2\pi\rho$, where $S = \pi(r_a^2 - r_p^2)$, $V = H \cdot S$, $H = 0{,}088$ AU. Comparison of the values ρ_2 shows that the critical value of density ρ_0 is estimated as $\rho_0 \approx 10^{-14}\ g/\mathrm{cm}^3$. If the gravitational instability condition $\rho > \rho_0$ will be supported by the influx of dust particles from the outer zone of the ring, in the ring will be formed only one planetezimal. If the flow of dust particles will stop, the seal ring is local and as a result of the compression ring will form several planetezimals.

Table 1: The spatial and surface density of the proto-planetary rings

Planets	ρ (g/cm^3)	ρ_1 (g/cm^2)	ρ_2
1.Mercury	$2{,}87\cdot10^{-12}$	6,237	$1{,}80\cdot10^{-11}$
2.Venus	$3{,}57\cdot10^{-10}$	36,96	$2{,}23\cdot10^{-9}$
3.Earth	$9{,}41\cdot10^{-11}$	36,41	$5{,}91\cdot10^{-11}$
4.Mars	$7{,}97\cdot10^{-13}$	1,374	$5{,}01\cdot10^{-12}$
5.Ceres	$5{,}46\cdot10^{-16}$	0,0006	$3{,}43\cdot10^{-15}$
6.Jupiter	$3{,}92\cdot10^{-10}$	138,5	$2{,}46\cdot10^{-9}$
7.Saturn	$2{,}99\cdot10^{-11}$	12,58	$1{,}88\cdot10^{-10}$
8.Uranus	$1{,}35\cdot10^{-12}$	0,444	$8{,}48\cdot10^{-12}$
9.Neptune	$3{,}82\cdot10^{-12}$	0,272	$2{,}40\cdot10^{-11}$
10.Pluto	$4{,}50\cdot10^{-16}$	0,001	$2{,}83\cdot10^{-15}$

3. Accumulation of dust particles in the proto-planetary rings

When the gravitational instability condition is satisfied, dust particles will combine to form the body of various mass and sizes. We assume that the dust particles of mass m_i are moved by the Kepler orbits around the central body of mass M ($M >> m_i$). Elliptical orbits of the particles in the polar coordinate system can be expressed as the following: $s = 1 / r = [1 + e\cdot\cos(v)]/p + G(M + m_i)/h^2$, where $p = a(1 - e^2)$, a, e, v - semi-major axis, eccentricity and the true anomaly of the particle orbits, h - the arbitrary constant. With the accumulation of dust particles orbit of new particles with mass $m = m_1 + +m_2 + ... + m_i$, will be changed. The equations of motion for new particles can be presented in the following form: $ds_i /dv_i + k_i^2 s_i = G(M + m_1 + m_2 + ... + m_i)/h^2$, $dv_i/dv = k_i$ [2], $i = 1, 2, ... , n$. The solutions of these equations are as follows:

$$s_i = (e/p)\cdot\cos(k_i v) + G(M + m_1 + ... + m_i)/(h^2 k_i^2). \quad (2)$$

New particles will have the following orbital elements: $p_i = h^2 k_i^2 /[G(M + m_1 + m_2 + ... + m_i)]$, $e_i=e\cdot p_i /p$. Since the mass of the dust particles and the newly formed particles is small compared with the central body, the changes in the orbital elements will also be small ($e_i \approx e$, $p_i \approx p$). However, the accumulation of orbital bodies will have a revolving of apsidal line. As a result, the process of accumulation will occur in a short period of time, which is comparable with the period of rotation of the apsidal line.

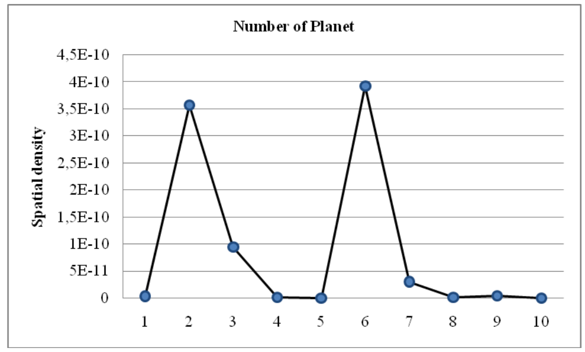

Figure 1: The spatial density of proto-planetary rings.

4. Summary and Conclusions

According to the model of accumulation (2), the orbits of the newly formed body will have a revolving line of apsides. In this case, the known anomalies of the perihelion motion of planets orbit are the result of accumulation. Moreover, if planetezimal formed by accumulation, according to the model (2), its orbit can not be a conic curve. The rotation of the line of apsides will ensure the rapid accumulation of particles in a single body with a spherically symmetric shape. As a result of the accretion of particles, most of the kinetic energy of the collision is converted into thermal energy. The remaining energy will generate a wave motion of the particles [2]. Compare the width, density and arrangement of proto-planetary rings shows that the presently observed distribution of the orbits of the planets is characteristic of wave action in the early stages of the evolution of the proto-planetary disk.

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