

## Research of a lunar surface with use the fractal analysis

A. Andreev (1), Y. Nefedyev (1), R. Zabbarova (1), K. Churkin (2)

(1) Engelhardt Astronomical Observatory, Russia (star1955@mail.ru / Fax: +7843-2927797)

(2) Kazan Federal University, Russia (sky0606@mail.ru / Fax: +7843-2927797)

### Abstract

A new method of comparative estimation of the reliable data of lunar maps is offered. The estimation is made by comparison of high-altitude lines using fractal analysis. The influence of the variances of the Moon's macrofigure is determined by the method of comparison of fractal dimensions.

### 1. Introduction

Research on the figure and rotation of the Moon including work on the construction of charts of its marginal zone by various methods is traditionally carried out at the Engelhardt Astronomical Observatory (EAO). In particular, this research is important for reduction in lunar occultations on the basis of which it is possible to solve a number of astrometric and astrophysical problems.

A number of factors greatly influence the accuracy of the results of these observations: the accuracy of recording the moment of occultation, the errors in the star coordinates, the accuracy of ephemeris position of the Moon and the accuracy of the charts of its marginal zone.

By now, highly accurate theories of the movement of the Moon have been developed and the coordinates of the stars have been determined on the basis of space measurements with an accuracy of several multiarcseconds. Therefore difficulty in the process of reduction in lunar occultations arises during accounting for the irregularities of its limb.

The existing charts of the marginal zone of the Moon have defects. Research on lunar maps of a

marginal zone is complicated. First of all, it concerns the reliability of the data given on the maps. To resolve this task, one method of research is the comparison of the structure of the high-altitude lines of data appropriate to identical lunar coordinates. However, such a comparison requires a large volume of calculations. It is known that to resolve the many problems in the description of a regional zone a macrofigure with a high accuracy is required.

### 2. Method of comparison of fractal dimensions

We researched the influence of the macrofigure of the Moon by the method of comparison of fractal dimensions. For this purpose we constructed detailed maps of the marginal zone of the Moon on the basis of data obtained from heliometric observations and taking into account the

TABLE I The Results of Estimation of the Fractal Dimensions of Maps N1 and N2

$P \text{ (deg)}$	$M_1$	$M_2$
0-45	1.42	1.39
45-90	1.34	1.34
90-135	1.44	1.36
135-180	1.30	1.32
180-225	1.40	1.34
225-270	1.37	1.31
270-315	1.33	1.79
315-360	1.30	1.37

first model of the figure of the Moon given by Jakovkin (maps N1) [3]. For comparison, maps N2 [2] were obtained. On the basis of observations obtained on the heliometer at EAO during 50 years, maps N1

and maps N2 are constructed. The charts contain isohypses of the marginal zone of the Moon extending over 10" on both sides of the mean position of the border line. In order to find by the aid of these charts the elevation of a given point of the limb above the mean level or to determine the irregularity of the limb, we have to compute, for the given position angle of this point (reckoned from the centre of the Moon's disc), two coordinates introduced by Hayn [1], namely P and D, where P is the selenocentric longitude reckoned along the mean limb from the north pole of the Moon, like the position angles, and D is the latitude taken positively for that part of the disc that is nearer to the observer. Thus the material for our research are of identical types. Firstly, segments of a lunar regional zone for every 45" on P were considered. For each segment, profiles of a surface for a certain D with a step of 2" were constructed. Thus 80 profiles were constructed. Secondly, the fractal dimension for each structure was defined according to the formula

$$d = \lim_{\sigma \rightarrow 0} \left( \frac{\ln[N(\sigma)]}{\ln(1/\sigma)} \right),$$

where  $d$  is the fractal dimension,  $\sigma$  is the length of the chord between the points of a profile and  $N$  is the number of chords. In the final analysis, Table I was obtained.

$P$  is the selenocentric longitude,  $MI$  is the mean fractal dimension for maps N1 and  $M2$  is the mean fractal dimension for maps N2.

### 3. Summary and conclusions

The results in Table I approximately correspond to the estimations that agree well between the mean fractal dimensions for maps N1 and N2. Our determinations of the fractal dimensions for maps N1 and N2 have made it possible to make some

plausible guesses about the readout of heights on the Moon's surface.

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