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Tensor equations in slightly aspherical configurations: get peace of mind with TenGSHui

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Abstract

TenGSHui is a Mathematica package to handle tensor equations in aspherical configurations. It is particularly suited to study the global dynamics of slightly flattened celestial bodies. The purpose of this contribution is to discuss a number of illustrative applications.

1. Introduction

Tensor equations are ubiquitous in classical physics: examples include the balance laws of fluid and continuum mechanics and thermodynamics, the flux and circulation laws of electromagnetism and gravitation, and the constitutive equations of materials. Numerous problems of geophysical or astrophysical relevance can be tackled by assuming that the Earth, the planet or the star is spherically symmetric. Usually, flattening effects are small and can be evaluated by perturbation methods. Some problems however critically depend on the shape of the body. For instance, irregularities in the Moon's spin rate (libration) and in the Earth's spin axis orientation (nutation) arise because of their elongated shape (equatorial and polar flattening), which leaves them exposed to the gravitational pull of celestial neighbours. Asphericity generally increases the technical complexity of numerical implementation, but even when spherical symmetry is applicable, it is convenient to leave these cumbersome, dull tensor manipulations to computers.

2. How it works

TenGSHui performs tensor operations on the basis of generalised spherical harmonics [1, 2], and maps aspherical domains to equivalent spherical domains [3] using an expansion scheme that can be carried up to the desired power of a small formal parameter ε (typically related to the flattening). This permits a systematic treatment of tensor equations in slightly aspherical domains. Besides, symbolic capabilities built in the programming language mean it is possible to format equations in usual mathematical notation. Therefore, TenGSHui should not be too difficult to acquaint with.

3. Applications

TenGSHui can be used to determine analytical solutions in simple spherically symmetric problems (e.g. free and forced oscillations – including tidal Love numbers – of a homogeneous compressible planet and of a layerwise homogeneous incompressible planet, magnetic induction in a planet with layerwise constant electrical conductivity). These solutions are described in various publications but with TenGSHui, you could independently get them in your own normalisations. The expansion scheme used in TenGSHui can also be checked on a number of problems that can be solved by alternative methods (e.g. gravitational potential of oblate spheroids where solutions can sometimes be found in oblate spheroidal coordinates, hydrostatic figure theory including Maclaurin spheroids). We will use TenGSHui to discuss a less straightforward problem: the libration of icy moons [4].

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