

A new analytical form for the magnetic power spectrum : application to the internal structure of giant planets

B. Langlais (1), H. Amit (1), H. Larnier (1,2), E. Thébault (3) and A. Mocquet (1)

(1) Laboratoire de Planétologie et Géodynamique, CNRS UMR 6112, Université de Nantes, 44322 Nantes cedex 3, France,

(2) Institut de Physique du Globe de Strasbourg, Université de Strasbourg, EOST, CNRS, Strasbourg, France, (3) Équipe de Géomagnétisme, Institut de Physique du Globe de Paris, CNRS UMR 7154, 75252 Paris cedex 5, France

Abstract

We propose two new analytical forms to the Mauersberger-Lowes geomagnetic field spectrum at the core-mantle boundary which can be used to determine the radius of the outer liquid core or more generally the radius of the electrically conductive and convective layer where the dynamo operates. We argue that two sub-families of the (geo)magnetic field exhibit flat spectra at the Earth's core-mantle boundary. The first is the non-zonal spectrum, the second is the spectrum associated with the field which is symmetric about the equator (the latter often termed the quadrupole family).

We first test our two analytical forms using two approaches on the geomagnetic field. We estimate at the seismic core radius the agreement between the actual spectrum and the theoretical one. We then estimate the magnetic core radius, i.e. the maximum depth from the Earth's surface at which the spectrum flattens. In both cases we show that the two sub-families offer a better agreement with the actual spectrum compared with previously proposed analytical expressions, while predicting a magnetic core radius within less than 10 km of the seismic core radius. These new analytical forms supersede all existing expressions to infer the core radius from magnetic field information because the low degree terms (and especially degree 1 term) are used. The fact that the two new forms converge and return the same dynamo radius can also be used to determine the maximum spherical harmonic truncation degree of the model.

Second we apply our new analytical forms to infer the radius of the dynamo regions on Jupiter, Saturn, Uranus and Neptune. We use the magnetic field models of Ridley [2012], Cao et al. [2011] and Holme and Bloxham [1996]. We give in Table 1 our estimates of the dynamo radii for given maximum truncation degrees. The axisymmetric nature of the magnetic field

of Saturn prevents the use of the non zonal form. For the three other planets both forms converge for N_{max} equal to 4 or 5. These new results offer independent constraints on the internal structure of these planets.

Table 1: Relative (with respect to planetary radii) dynamo radius estimates using the Non Zonal (NZ) and the Quadrupole Family (QF) approaches for given maximum spherical harmonic degree N_{max} .

| Planet | N_{max} | NZ | QF |
|---------|-----------|------|------|
| Jupiter | 5 | 0.88 | 0.89 |
| Saturn | 5 | na | 0.23 |
| Uranus | 4 | 0.66 | 0.66 |
| Neptune | 5 | 0.92 | 0.93 |

References

H. Cao, C. T. Russell, J. Wicht, U. R. Christensen, and M. K. Dougherty. Saturn's high degree magnetic moments: Evidence for a unique planetary dynamo. *Icarus*, 221:388–394, 2011.

R. Holme and J. Bloxham. The magnetic fields of Uranus and Neptune: Methods and models. *J. Geophys. Res.*, 101:2177–2200, 1996.

V. A. Ridley. *Jovimagnetic Secular Variation*. PhD thesis, The University of Liverpool, 2012.