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Application of an Evolution Strategy in Celestial Mechanics

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Abstract

As an alternative to classical deterministic gradient searches, evolutionary strategies (ES) can be used to implement technical optimization algorithms for a wide range of problems. They are universal, undemanding, robust, easy to implement, and can be considered as a compromise between volume and path orientated searches for the optimal solution.

It is demonstrated that ES are applicable to inverse or boundary value problems: e.g. determination of gravity field coefficients, or the determination of orbital elements from given position vectors.

1. Introduction

In general, evolutionary algorithms comprise the two branches genetic algorithms [1] and evolution strategies [4], both of which were invented in the late 1960s and early 1970s. Here we focus solely on ES which have seen many improvements within the last decades and can now be regarded as a real alternative to standard optimization techniques in many areas, especially in cases where gradient methods like the classical least-squares algorithm fail.

Compared to other optimization techniques, ES are easy to adapt to various problems, because one rarely needs any a priori insight into the mathematical or physical nature of the optimization task. Once implemented, the same algorithm can be applied to a wide range of problems without substantial changes. The only necessary condition for ES to successfully operate on a given specific problem is the inherent existence of strong causality, which here means that similar causes lead to similar results, i.e., there is no (short-term) chaotic behavior in the underlying system.

In the following sections we present 2 examples for the application of an ES with covariance matrix adaptation (ES-CMA) [2]. Any of the strategy parameters were chosen empirically here. This could be avoided by the implementation of a Meta-ES, that eventually can, in addition to the problem specific unknowns, optimize its own strategy parameters automatically.

2. Gravity field determination

The goal is to find spherical harmonic coefficients c_{nm} and s_{nm} up to a given maximum degree $n_{\max}$ and order $m_{\max}$, representing an $n \times m$ gravity field of a central body, e.g. Earth. Here we solve for a 4×4 gravity field, equivalent to a 21-dimensional optimization problem.

Earth's gravity field directly influences the motion of an orbiting satellite, so we can treat the latter as a test mass. In order to determine the coefficients, several satellite positions (simulated in this study) are given. We search for an optimal set of spherical harmonics, leading to calculated positions $\mathbf{r}_i^c$. Comparing them with the simulated vectors $\mathbf{r}_i^s$ yields deviations $\Delta\mathbf{r}_i = \mathbf{r}_i^s - \mathbf{r}_i^c$. These differences should not exceed a chosen threshold value.

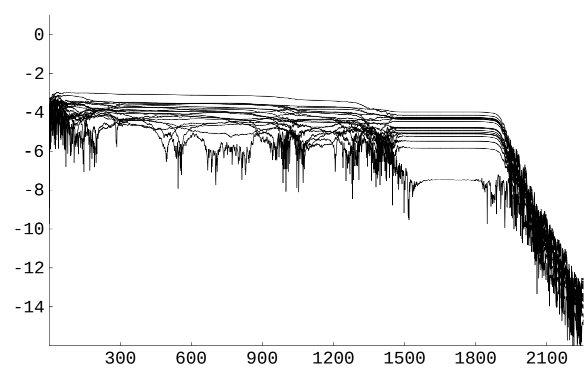

Figure 1: Logarithm of the absolute residual values of the unknowns vs. generation number.

Depending on the norm, the performance index (objective function, quality criterion) may be defined as $Q = \sum_{i=1}^{N} \|\Delta\mathbf{r}_i\| \rightarrow \min$, where N is the number of given satellite positions. The termination quality was set to $Q^* = 1/1000\, mm$, and for $N = 90$ a (1,40)-ES-CMA was realized. The values in the round bracket indicate that for each new generation there is 1 parent creating an offspring of 40 individuals, and *only* the (mutated) offspring is subject to selection afterwards. Figure 1 illustrates the evolution of the unknowns, and table 1 provides their final values.

Table 1: Final result of the ES optimization, all values in 10^{-10}. Digits identical with the original spherical harmonics (used in the simulation) are in bold print.

n	m	c_{nm}	s_{nm}
2	0	$-\mathbf{4841695.48}34480$	-
3	0	$+\mathbf{9571.7060}002975$	-
4	0	$+\mathbf{5397.770}5833457$	-
2	1	$-\mathbf{1.869}4714700433$	$+\mathbf{11.95}4500954474$
3	1	$+\mathbf{20301.372}076698$	$+\mathbf{2481.3079}540691$
4	1	$-\mathbf{5362.435}8305647$	$-\mathbf{4737.72}49759825$
2	2	$+\mathbf{24392.60}9849473$	$-\mathbf{14002.66}5205972$
3	2	$+\mathbf{9047.063}6114776$	$-\mathbf{6189.228}5463862$
4	2	$+\mathbf{3506.701}2168619$	$+\mathbf{6625.713}6424735$
3	3	$+\mathbf{7211.449}1711647$	$+\mathbf{14142.039}502771$
4	3	$+\mathbf{9908.68}82512345$	$-\mathbf{2009.87}46087090$
4	4	$-\mathbf{1884.81}46556533$	$+\mathbf{3088.48}15006772$

3. Satellite orbit from two positions

The goal is to find the solution to a seemingly simple boundary value problem. Given are position vectors $\mathbf{r}_A$, $\mathbf{r}_B$, valid at epochs t_A, t_B with $t_B > t_A$ (to fix the sense of direction for the satellite's motion), and a known force field, e.g. a 8×8-gravity field, cf. figure 2.

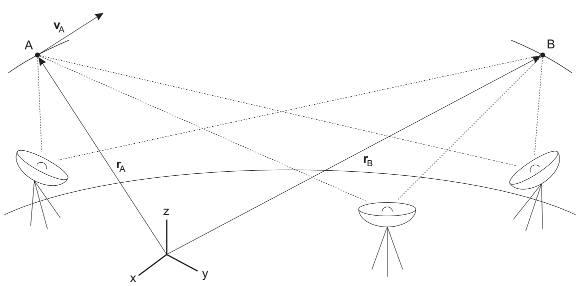

Figure 2: Exemplary boundary value problem.

The fundamental task is to transform the original boundary value problem into an initial value problem, i.e., search for the corresponding initial velocity vector $\mathbf{v}_A$. Knowing the initial state vector $\mathbf{z}_A := (\mathbf{r}_A, \mathbf{v}_A)^T$, and the arc length, i. e., the time of flight $t_B - t_A$, the satellite orbit between A and B can then be determined via traditional methods. There exist only three unknowns, namely the cartesian components of $\mathbf{v}_A$.

It is absolutely sufficient to be familiar with the equation of motion of the perturbed two-body problem and its numerical integration (NI). We do not require any other theoretical knowledge about celestial mechanics, e.g. the availability of integrals of motion. As a simple performance index one can define $Q = \|\Delta\mathbf{r}_B\| = \|\mathbf{r}_B - \mathbf{r}_B^{NI}\| \rightarrow$ min.

Again, the (1,40)-ES-CMA was employed. The termination quality was set to $Q^* = 1 \cdot 10^{-10}$ mm. For a given numerical example, a solution was found after 145 generations, see figure 3 for the quality plot.

Figure 3: Quality in km vs. generation number.

4. Summary and Conclusions

Only 2 applications of evolutionary strategies in celestial mechanics were presented here. In future, this optimization technique should gain more importance, especially when it comes to the direct solution of inverse problems. The ever improving hardware and software capabilities support this direct approach. The author plans to use ES for improved asteroid modeling within the calculation of a new solar system ephemeris [3].

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