

## Method of the star catalogues analysis on the basis of occultation

Y. Nefedyev (1), N.Varaksina (1), N.Petrova (1), K. Churkin (1), R. Zabbarova (1), A. Andreev (1)  
 (1) Engelhardt Astronomical Observatory Kazan Federal University, Russia (star1955@mail.ru / Fax: +7843-2927797)

### Abstract

In this abstract are discussed the algorithm of determining of the orientation of the system of coordinates in the Hipparcos catalogue relative to the dynamic one for reduction of lunar occultations.

### 1. Introduction

In recent years, the implementation of the inertial celestial coordinate system achieved great success. Cosmic catalogue Hipparcos was obtained with millisecond accuracy and realizing the inertial system. To study the orientation of the HIPPARCOS catalog on with respect to the dynamic coordinate system were used 48 observations of small planets. The disadvantage of this method is the poor conditioning of the system of conditional equations, which greatly affects the results of the decision by the method of least squares (OLS). We propose the use of an alternative method based on long-term records occultations of stars by the Moon and does not have the drawbacks of the method using observations of minor planets.

### 2. Algorithm of determining of the orientation of the system of coordinates

The algorithm based on the lunar occultations.

1. Transformation of star coordinates from the system of initial catalogue into the system of Hipparcos catalogue and calculation of apparent, topocentric differences between  $\alpha, \delta$  of the occulted star and coordinates of the center of the Moon mass.

$$\begin{aligned} (\Delta\alpha_{o-c} \cos \delta)_v &= (R_o - R_c) \sin \theta^*, \\ (\Delta\delta_{o-c})_v &= (R_o - R_c) \cos \theta^*. \end{aligned} \quad (1)$$

2. Calculation of constituents of topocentric optical libration of the Moon considering influence of physical libration of the Moon (PhLL) -  $I, b, C$  by the appropriate formulas (The Astronomical Almanac, 1984). In this case corrections for physical libration

3. Calculation of values ( $O-C$ ) with  $\alpha$  and  $\delta$  considering the corrections from the charts of Moon marginal zone. In the (1) values  $(\Delta\alpha_{o-c} \cos \delta)_v$ ,

$(\Delta\delta_{o-c})_v$  are differences of appropriate star coordinates and the center of Moon's mass without considering heights of Moon marginal zone. From  $\alpha$  and  $\delta$  constituents of heights of points of the Moon marginal zone above an average sphere of the lunar figure and topocentric lunar radius  $R_{moon}$  are

$$\begin{aligned} (\Delta\alpha \cos \delta)_h' &= (\Delta h' + R'_{moon}) \sin \theta^*, \\ (\Delta\delta)_h' &= (\Delta h' + R'_{moon}) \cos \theta^*, \end{aligned} \quad (2)$$

according to maps of Moon marginal zone, where  $\theta^*$

- positional angle of the star for projections of the moon centre of the masses on celestial sphere. Then the apparent values ( $O-C$ ) on the moment of the observation will be

$$\begin{aligned} (\Delta\alpha_{o-c} \cos \delta)_v' &= (\Delta\alpha \cos \delta)' - (\Delta\alpha \cos \delta)_h', \\ (\Delta\delta_{o-c})_h' &= (\Delta\delta)' - (\Delta\delta)_h', \end{aligned} \quad (3)$$

We reduce them to the average distance from the Earth to the Moon

$$\begin{aligned} (\Delta\alpha_{o-c} \cos \delta)_m &= (\Delta\alpha_{o-c} \cos \delta)_v' \times (932.58'') / R', \\ (\Delta\delta_{o-c})_m &= (\Delta\delta_{o-c})_h' \times (932.58'') / R' \end{aligned} \quad (4)$$

4. First way of the determination of the orientation of the system of coordinates there is: Let orientation of the catalogue system of coordinates relative to dynamic one be specified by angles of rotation  $\epsilon_x, \epsilon_y, \epsilon_z$  around  $x, y, z$  - axes of dynamic system of coordinates, we have following equations which are fair for the small rotations [4]:

$$\begin{aligned} \Delta\alpha_{o-c} \cos \delta &= \sin \delta \cos \alpha \epsilon_x \\ &+ \sin \delta \sin \alpha \epsilon_y - \cos \delta \epsilon_z, \\ \Delta\delta_{o-c} &= -\sin \alpha \epsilon_x + \cos \alpha \epsilon_y. \end{aligned} \quad (5)$$

They can be used for calculation of angles of rotation  $\epsilon_x, \epsilon_y, \epsilon_z$  of catalogue system of coordinates relative to the dynamic one. Further, since

$$\epsilon_x = -\Delta\varepsilon, \epsilon_y = \Delta L \sin \varepsilon, \epsilon_z = \Delta A - \Delta L \cos \varepsilon, \quad (6)$$

we obtain  $n$  conditional equations for determination enumerated below adjustments, where  $n$  - numbers of the observations.

$$\begin{aligned} \Delta\alpha_{o-c} &= -\Delta A + \\ &+ \Delta L \cos \varepsilon (1 + \tan \varepsilon \tan \delta \sin \alpha) - \Delta \varepsilon \tan \delta \cos \alpha, \\ \Delta\delta_{o-c} &= -\Delta D + \Delta L \sin \varepsilon \cos \alpha + \Delta \varepsilon \sin \alpha. \end{aligned} \quad (7)$$

where  $\Delta A, \Delta D$  - corrections to the equinox and the equator of the catalogue,  $\Delta L, \Delta \varepsilon$  - corrections to the Moon longitude and inclination of ecliptic to the equator.

The rates of change of angles  $\epsilon_x, \epsilon_y, \epsilon_z$  are determined by the following equations

$$\begin{aligned} w_x &= -\Delta \dot{\varepsilon}, w_y = \Delta \dot{L} \sin \varepsilon, \\ w_z &= \Delta \dot{A} - \Delta \dot{L} \cos \varepsilon, \end{aligned} \quad (8)$$

where the points indicate rates of change of appropriate values. The angles of rotation  $\dot{\epsilon}_{xt}, \dot{\epsilon}_{yt}, \dot{\epsilon}_{zt}$  for time there is [1]:

$$\begin{aligned}\dot{\epsilon}_{xt} &= \epsilon_x + w_x(t - t_0), \dot{\epsilon}_{yt} = \epsilon_y + w_y(t - t_0), \\ \dot{\epsilon}_{zt} &= \epsilon_z + w_z(t - t_0).\end{aligned}\quad (9)$$

Thus, letting (6), (8) and (9) in equations (7), it reduces to the form:

$$\begin{aligned}\Delta\alpha_{o-c} &= -(\Delta A + \Delta\dot{A}(t - t_0)) \\ &+ (\Delta L + \Delta\dot{L}(t - t_0)) \cos \epsilon (1 + \tan \epsilon \tan \delta \sin \alpha) - \\ &- (\Delta \epsilon + \Delta\dot{\epsilon}(t - t_0)) \tan \delta \cos \alpha, \\ \Delta\delta_{o-c} &= -\Delta D + (\Delta L + \Delta\dot{L}(t - t_0)) \sin \epsilon \cos \alpha \\ &+ (\Delta \epsilon + \Delta\dot{\epsilon}(t - t_0)) \sin \alpha.\end{aligned}\quad (10)$$

The relations equations  $(\Delta\alpha_{o-c} \cos \delta)_m$  and  $(\Delta\delta_{o-c})_m$  is  $(\alpha_{cat} - \alpha_{dyn})$  and  $(\delta_{cat} - \delta_{dyn})$ . Thus, substituting these values into left-hand sides of equations (10), we receive a system with 2n conditional equations of form (10). Let's solve them by the method of least squares and determine required angles of orientation of the system of coordinates of the Hipparcos catalogue relative to dynamic system of coordinates  $\epsilon_x, \epsilon_y, \epsilon_z, w_x, w_y, w_z$  taking into consideration equations (6) and (8);

5. Second way of the determination of the orientation of the system of coordinates there is: Let, there is [3]:

$$\Delta\sigma = \sum_{k=1}^N \frac{\partial\sigma}{\partial Q_k} \Delta Q_k, \quad (11)$$

where  $\sigma$  - angular distance from a star to ephemeris center of the Moon's mass at the moment of occultation,  $\Delta\sigma$  is  $(O-C)_\sigma$ ,  $\frac{\partial\sigma}{\partial Q_k}$  known factors at

the corrections to parameters under study-  $Q_k$ ,  $N$  - number of parameters. At present time, lunar and planet ephemerides have higher accuracy, and the positions of stars were determined with accuracy of millisecond in the Hipparcos catalogue. Thus, the number of corrections in question can be minimized. To a first approximation, it was decided to decrease  $N$  to 3 in conditional equations (11) and to find the corrections only to some parameters of the theory of the Moon's movement, to the origin of the right ascension of the Hipparcos catalogue, to inclination of equator to ecliptic, to the system of coordinates of charts of the Moon marginal zone and probably, to the equator of the Hipparcos catalogue.

In these conditions, the conditional equations (11) will take the form [5]

$$\Delta\sigma = \frac{\partial\sigma}{\partial\lambda} \Delta\lambda + \frac{\partial\sigma}{\partial\alpha_0} \Delta\alpha_0 + \frac{\partial\sigma}{\partial\epsilon} \Delta\epsilon, \quad (12)$$

Here  $\Delta\lambda = \Delta w_1^0$ ;  $\Delta\epsilon = \Delta\epsilon_0$ ;

$$\Delta\alpha_0 = (\Delta A + \Delta\alpha_s \sin \alpha_{cat} + \Delta\alpha_c \cos \alpha_{cat}) \\ - (\Delta R_0 + \Delta R_{1c} \cos \theta_* + \Delta R_{2s} \sin 2\theta_* + \Delta R_{2c} \cos 2\theta_*);$$

$w_1^0$  - constant member of the Moon's mean longitude;  $(\Delta A + \Delta\alpha_s \sin \alpha_{cat} + \Delta\alpha_c \cos \alpha_{cat})$  - constant correction to equinox of a catalogue and variables dependent on  $\sin \alpha_{cat}$  and  $\cos \alpha_{cat}$  corrections to the right ascension of a catalogue;  $(\Delta R_0 + \Delta R_{1c} \cos \theta_* + \Delta R_{2s} \sin 2\theta_* + \Delta R_{2c} \cos 2\theta_*)$  - constant and variable corrections to the system of coordinates of the charts of the Moon marginal zone;  $\epsilon_0$  - constant member of inclination of the equator to the ecliptic;  $\Delta\alpha_s$  - systematic correction in right ascension of Hipparcos varying as  $\sin \alpha$ ;  $\Delta\alpha_c$  - systematic correction in right ascension of Hipparcos varying as  $\cos \alpha$ ;  $\Delta R_{1c}$  - latitude component of shift of center of maps lunar marginal zone datum;  $\Delta R_{2s}$  - longitude component of correction to ellipticity of maps lunar marginal zone datum; latitude component of correction to ellipticity of maps lunar marginal zone datum. The solution of conditional equations (12) will be found by the method of the least squares through the iteration, that is, at first the worst corrections will be determined, then after their account there will be found other required values. To put otherwise, the solution will be found through successive approximations.

### 3. Summary and Conclusions

Thus, algorithm of reduction of lunar occultation with the aim to estimate the accuracy of orientation of the space system of coordinates was constructed by several methods. The using of lunar occultations will allow obtaining more authentic information about orientation of the system of coordinates of the Hipparcos catalogue. Work was supported by grants RFBR 13-02-00792-a, 12-02-97000-reg-a, 14-02-31296-mol-a and 14-02-92113 Russian - Japanese - a

### References

- [1] Batrakov Yu.V., Chernetko Yu.A., et al. (1999) Astron. & Astrophys. 352, 703.
- [2] Eckhardt D.H. (1981) M&P, 25, 3.
- [3] Morrison L.V. (1979) MNRAS 187, 41.
- [4] Nefedjev Yu.A., Nefedjeva A.I. and Borovskikh V.S. (2002) Kazan: Unipress, 1.
- [5] Soma M. (1985) Cel. Mech. 35, 45.