

Phobos' Rotational Elements From Control Point Network Analysis: A New Analytic Approach

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Abstract

We focus on the Martian satellite Phobos and present a modified approach to control point network analysis and rotation studies. In particular, we analytically solve for the pole axis orientation, precession, and longitudinal librations, a capability not previously implemented in our least-squares bundle block adjustment. Our results confirm the previously adopted pole axis orientation of Phobos and confirm the forced libration amplitude of -0.78 degrees reported by [1].

1. Introduction

The orientation of a planetary body is commonly described by its pole axis orientation and the relative position of the prime meridian with respect to the vernal equinox at a specific time epoch. For Phobos, [1] recommend a parameter set that describes a precession with a period of about 790 days and a forced libration (small oscillation superimposed on the mean rotation rate) caused by the interaction of Phobos' body with the Martian gravity field.

Planetary reference frames are typically realized by control point network analyses. The orientation parameters of planetary bodies are, however, not easily determined as the available software tools for the analysis operate in the body-fixed reference frame. Hence, previously, the parameters have been determined through empirical approaches, in particular by systematically scanning through the parameter space e.g. [2]. Not surprisingly, some of the rotational parameters of Phobos, such as rotational axis orientation or precession have never been directly measured.

In [3] is already demonstrated that an analytical determination of the rotational elements based on a least-squares bundle block adjustment can be successful even with a data set covering a short time

period. We present a modified approach, in which we determine directly the rotation parameters in inertial space for which we developed a new implementation of the bundle block adjustment.

For verification of the chosen approach to analytically determine the rotational parameter images of the Super Resolution Camera (SRC) on board the European Mars Express (MEX) spacecraft and the Viking Orbiter (VO) camera were used. The data set consists of measured image coordinates of the tie points that had already been used to derive earlier versions of the control point network [4]. The SRC images were obtained between May 18, 2004 and Aug 13, 2007 thus covering the period of the currently adopted precession 1.5 times. A more important aspect of this analysis is to verify and increase accuracies, since already small changes of the derived amplitude subsequently change, in combination with other observations, the mass distribution models (as discussed in [5, 6]).

2. Forced Libration Amplitude

Results of efforts to determine the forced libration amplitude for Phobos differ significantly. Observations range from 0.79° [7] to 1.24° [4] while models assuming a homogeneous mass distributions suggest an amplitude between 0.81° [8], 1.1° [4] and 1.99° [9].

3. Least-Squares Adjustment

A novelty of our approach is the formulation of the adjustment problem in the inertial framework. The function to describe the image observations is derived from

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{Phobos} = R_2 * R_1 * \begin{pmatrix} x \\ y \\ f \end{pmatrix}_{Camera} + \begin{pmatrix} Xo \\ Yo \\ Zo \end{pmatrix}_{Phobos}$$

$R_{1,2}$ are rotation matrices. R_1 rotates from the frame of the image coordinates (x,y,f) into the inertial

frame J2000 and R_2 rotates from J2000 frame to the IAU_Phobos frame. Apart from the classical six unknown parameters per frame in a bundle block adjustment, the Jacobi-matrix in our approach contains additionally the derivatives of the three parameters α , δ and ω which form the rotational matrix R_2 and are defined as the time-dependent functions

$$\begin{aligned}\alpha(t) &= \alpha_0 + a \cdot \cos(M1) - 0.108 T \\ \delta(t) &= \delta_0 + b \cdot \sin(M1) - 0.061 T \\ \omega(t) &= \omega_0 + c \cdot \sin(M2) + d \cdot \sin(M1) + k(t).\end{aligned}\quad (1)$$

In section 4 we give some of the values before and after adjustment, see [1] for explicit definition of the remaining constants (for a given time epoch t).

In a first approach the derivatives $d\alpha$ (resp. $d\delta$, $d\omega$) are built with respect to the start value α_0 (resp. δ_0 , ω_0). In a second adjustment we use the updated start values in the definition of (1) and build the derivatives $d\alpha$, $d\delta$, $d\omega$ with respect to the amplitudes a , b and c in equation (1). Note that d is left unchanged. c is the forced libration amplitude.

It should also be mentioned that all classical frame parameters were included as additional observations with weights derived from a prior classical bundle block adjustment that confirms the results from [4].

4. Summary and Conclusions

The original control network contained 689 points from which 680 remained after the new adjustment. The mean error of their variance-vector could be reduced to 19.14m (30m in [4] with 665 points).

Table 1: Pre- and post-adjustment values for Phobos orientation parameters.

Value	Apriori	Aposteriori
α_0	317.68	$317.0597 \pm 2.7e-04$
δ_0	52.90	$52.8998 \pm 1.77e-04$
ω_0	35.06	$35.0598 \pm 2.50e-04$
Amplitudes of time-varying parameters (cf. eq. 1)		
a	1.79	$1.79004 \pm 4.4e-05$
b	-1.078	$-1.08021 \pm 2.1e-04$
c	-0.78	$-0.78037 \pm 3.7e-04$

The determined values for the orientation of Phobos do not differ significantly from those previously

adopted. This is somewhat unexpected in comparison to the differing observations of the forced libration for Phobos. The current result could be a confirmation of the forced libration of -0.78 degrees but it is more likely that remaining stresses in the network are transferred to the orientations of the camera view vectors rather than to the fixed body orientation. This would have a greater effect than a slight rotation of the reference frame. We will further investigate this issue and report detailed results during the meeting.

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