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Tidal dissipation in heterogeneous bodies: Maxwell vs Andrade rheology

M. Běhounková (1) and O. Čadek (1)
(1) Charles University in Prague, Faculty of Mathematics and Physics, Department of Geophysics, Czech Republic
(marie.behounkova@mff.cuni.cz)

Abstract

The tremendous volcanism on Jupiter's moon Io as well as the huge activity at the south pole of Saturn's moon Enceladus show that tidal dissipation is a very strong source of energy for some bodies in the Solar System. Outside the Solar System, tidal heating in short-period exoplanets may cause Io-like volcanism, large-scale melting and even thermal runaways [1–4]. Here we further develop the method to compute tidal heating in heterogeneous bodies [5]. Especially, we concentrate on the Andrade rheology implementation. We study the impact of the improved model on bodies with large lateral viscosity variation such as Enceladus and tidally locked exoEarth with a large surface temperature contrast due to uneven insolation [6]. We discuss the influence of empirical parameters describing the Andrade rheology and compare the tidal heating and tidal stress obtained for the Andrade rheology with frequently used Maxwell models for different forcing frequencies.

1. Governing equations

In order to evaluate the stress and deformation due to tides, we consider small deformations in a hydrostatically pre-stressed spherical incompressible viscoelastic body. The time evolution of stress and displacement in such a body is governed by the mass and momentum conservation equations:

$$\nabla \cdot \boldsymbol{u} = 0, \quad (1)$$

$$-\nabla p + \nabla \cdot \boldsymbol{\sigma} = \rho \left(\nabla \Phi + \nabla V \right), \quad (2)$$

where p is the dynamic pressure, $\boldsymbol{u}$ is the displacement, $\boldsymbol{\sigma}$ is the deviatoric part of stress tensor, ρ is the density, Φ describes the time varying tidal potential and V is the perturbation of the gravitational potential induced by deformations of the surface. The relationship between the strain tensor $\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla \boldsymbol{u} + \nabla^{\mathrm{T}} \boldsymbol{u}\right)$ and the deviatoric stress is described by the Andrade rheology [7] which for stress-free initial conditions $\boldsymbol{\sigma}(t < 0) = \mathbf{0}$ reads

$$2\boldsymbol{\varepsilon} - \frac{\boldsymbol{\sigma}}{\mu} = \int_0^t \frac{\boldsymbol{\sigma}(t')}{\eta} \mathrm{d}t' + \int_0^t \mu^{\alpha-1} \frac{(t-t')^\alpha}{(\zeta\eta)^\alpha} \dot{\boldsymbol{\sigma}} \mathrm{d}t'. \quad (3)$$

$\tau_{\mathrm{M}} = \frac{\eta}{\mu}$ is the Maxwell time and α and ζ are empirical parameters. Besides the parameters needed to describe the Maxwell rheology, namely viscosity η and shear modulus μ, the Andrade rheology depend also on two additional parameters α and ζ.

We solve the equations in a spherical shell corresponding to either a mantle of terrestrial planets or a shell of icy moons. On the surface, a force equilibrium is prescribed. The boundary conditions on the bottom boundary combine the force equilibrium describing solid/liquid interface and no-slip boundary conditions for solid/solid interface.

2. Method

The viscoelastic response of a body to the tidal loading is solved numerically. Following method [5], we integrate the set of equations in the time domain until we reach a converge solution. This approach allows to introduce the lateral variation of viscosity as well as combination of boundary conditions on the bottom boundary. This method was originally developed for the Maxwell rheology, which, however, underestimates the dissipation rate for forcing periods shorter than the Maxwell time. In contrast, the Andrade rheology is designed to explain the viscoelastic response for periods corresponding to tidal forcing [8].

The time discretization of equations (1)–(3) at the time step $i+1$, considering constant time step Δt, is as follows:

$$\nabla \cdot \boldsymbol{u}_{i+1} = 0 \quad (4)$$

$$-\nabla p_{i+1} + \nabla \cdot \boldsymbol{\sigma}_{i+1} = \rho \left(\nabla \Phi_{i+1} + \nabla V_i \right), \quad (5)$$

$$2\boldsymbol{\varepsilon}_{i+1} - \frac{\boldsymbol{\sigma}_{i+1}}{\mu} = \sum_{j=0}^{j=i} \frac{\sigma_j}{\eta} \Delta t$$

$$+ \sum_{j=0}^{j=i-1} w_{ij} \left(\frac{\boldsymbol{\sigma}_{j+1}}{(\zeta\eta)^{\alpha}} - \frac{\boldsymbol{\sigma}_{j-1}}{(\zeta\eta)^{\alpha}} \right), \quad (6)$$

where w_{ij} are weights

$$w_{ij} = {}^{1}\!/\!{}_{2}(i-j+1)^{\alpha} \Delta t^{\alpha} \mu^{\alpha-1}. \quad (7)$$

The first term on the right-hand side of equation (6) is identical to the memory term in the case of a Maxwell body [9]. The second term on the right-hand side corresponds to the continuous distribution of relaxation processes, specific for the Andrade rheology, and it has to be evaluated at each time step from scratch as the weight w_{ij} changes with time. The evaluation of this term is thus time-consuming and memory-demanding as it requires to store the stress during the whole time evolution. Fortunately, it can be efficiently parallelized using MPI protocol.

The spatial discretization employs spherical harmonic function in horizontal directions and finite difference scheme in vertical direction [9]. The implementation of the Andrade rheology was carefully tested for the case of radially dependent viscosity against the traditional Laplace domain solution [10].

3. Applications

The tidal dissipation plays an important role in both internal dynamics and orbital evolution of many bodies within the Solar System, such as Io and Enceladus. Outside the Solar System, the short-period planets orbiting the parent star at close distance are strongly influenced by tidal heating. The tidal stress may also influence the tectonic activity as in the case of Enceladus [12] and its analysis may help to constrain the internal structure of the body [13].

The above described method is especially important for bodies with large lateral variations in viscosity. Here we concentrate on Enceladus the dichotomy of which suggests strong lateral variations within the ice shell. We compare the tidal heating and tidal stress within the ice shell obtained using the Maxwell and the Andrade rheology.

Tidally locked exoplanets exhibit a large surface temperature contrast between sub-stellar and anti-stellar sides due to uneven illumination of their surface if no atmosphere is present [6]. The surface temperature contrast induces asymmetric degree-1 pattern of mantle convection [14, 15] and large temperature and hence viscosity anomalies. Here we discuss the stress and tidal heating distribution in short-period exoEarths. We show the difference between the Maxwell rheology and the Andrade rheology for different frequencies. We concentrate on influence of empirical parameters α and ζ describing the continuous distribution of the relaxation processes. Further we study the time dependence of phase lag and time lag during one orbit.

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