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Modelling of Resonantly Forced Density Waves in Dense Planetary Rings

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Abstract

Density wave theory, originally proposed to explain the spiral structure of galactic disks, has been applied to explain parts of the complex sub-structure in Saturn's rings, such as the wavetrains excited at the inner Lindblad resonances (ILR) of various satellites.
The linear theory for the excitation and damping of density waves in Saturn's rings is fairly well developed (e.g. Goldreich & Tremaine [1979]; Shu [1984]). However, it fails to describe certain aspects of the observed waves. The non-applicability of the linear theory is already indicated by the "cusplike" shape of many of the observed wave profiles. This is a typical nonlinear feature which is also present in overstability wavetrains (Schmidt & Salo [2003]; Latter & Ogilvie [2010]). In particular, it turns out that the detailed damping mechanism, as well as the role of different nonlinear effects on the propagation of density waves remain intransparent.
First attemps are being made to investigate the excitation and propagation of nonlinear density waves within a hydrodynamical formalism, which is also the natural formalism for describing linear density waves.
A simple weakly nonlinear model, derived from a multiple-scale expansion of the hydrodynamic equations, is presented. This model describes the damping of "free" spiral density waves in a vertically integrated fluid disk with density dependent transport coefficients, where the effects of the hydrodynamic nonlinearities are included.
The model predicts that density waves are linearly unstable in a ring region where the conditions for viscous overstability are met, which translates to a steep dependence of the shear viscosity with respect to the disk's surface density. The possibility that this dependence could lead to a growth of density waves with increasing distance from the resonance, was already mentioned in Goldreich & Tremaine [1978]. Sufficiently far away from the ILR, the surface density perturbation caused by the wave, is predicted to saturate to a constant value due to the effects of nonlinear viscous damping. A qualitatively similar behaviour has also been predicted for the damping of nonlinear density waves, as described within a streamline formalism (Borderies, Goldreich & Tremaine [1985]).
The damping lengths which follow from the weakly nonlinear model depend more or less strongly on a set of different input parameters, such as the viscosity and the surface density of the unperturbed ring state. Further, they depend on the wave's amplitude at resonance. For a real wave, which has been excited by an external satellite, this amplitude can be deduced from the magnitude of the satellite's forcing potential.
Appart from that, hydrodynamical simulations are being developed to study the nonlinear damping of resonantly forced density waves.

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