

# Relativistic Effects for the JUICE On-Board Clock

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## Abstract

We studied relativistic effects on spacecraft clock rates for ESA's JUICE mission and derive a relationship between Dynamical Time TDB and On-board Time. We analysed the S/C On-Board clock rate by using the JUICE Mission nominal trajectory. We identify significant changes in the rate due to the various spacecraft operational phases. The offset in time between Dynamical Time TDB and On-Board clock may run up to  $\approx 3.22$  seconds at the end of the mission after  $\approx 11.5$  years due to relativistic effects.

## 1. Introduction

Accurate timing is critical for deep space missions since events that did happen have to be tagged correctly or commands have to be executed at the right time, e.g., for labelling when a certain measurement was taken or when a maneuver has to be executed so that the SC follows its planned trajectory.

For certain applications one needs quite precise timing, as for example with the application of range measurements from Ground Stations to the S/C. Since one wants to relate events happening at different locations in the Solar System with each other, the formulation of the equations and the included effects has to be quite precise.

## 2. Influence of Relativistic Effects on S/C Clocks

Clocks are the crucial elements for measuring the timing and are influenced by various effects, which change their rate and therefore the timing itself. Besides stress e.g. from acceleration or vibration, change in pressure and humidity, hardware aging, electric or magnetic fields radiation and temperature [ 1 ] one can observe variations due to relativistic effects. This becomes important if one wants to relate events at different locations in the solar system with each other. Changes in the rate of clocks with respect to another reference frame occur due to change of potential at and velocity of the clock. Following

Moyer [ 2 ] one can describe the change of the rate of a clock by

$$\frac{d t_{CLOCK}}{d t_{TCB}} = \left[ 1 - \frac{2\varphi}{c^2} - \left( \frac{v}{c} \right)^2 \right]^{\frac{1}{2}} \quad (1)$$

with  $\varphi$  being defined as

$$\varphi = \sum \frac{\mu_i}{r_i} = \sum \frac{G \cdot m_i}{r_i} \quad (2)$$

Thereby TCB (Barycentric Coordinate Time) represents a Coordinate Time one would measure with an atomic clock in an inertial frame located at the SSB (Solar System Barycenter).  $\varphi$  is the potential at the clocks location, summed up over all bodies that are taken into account, with their gravitational parameters  $\mu_i = G \cdot m_i$ . Thereby  $r_i$  is the distance between the clock and a certain body. While  $c$  is the speed of light  $v$  represents the velocity of the clock with respect to SSB.

The Dynamical Time TDB (Barycentric Dynamical Time) is the standard for planetary ephemerides and with respect to TCB defined as

$$\frac{d t_{TDB}}{d t_{TCB}} = 1 - L_b = 1 - 1.550519768 \cdot 10^{-8}. \quad (3)$$

Due to this definition, Earth based Coordinate Time systems, as for example TDT/TT or TCG, only exhibit periodic variations with respect to TDB. By combining Equation (1), (2) and (3), one can derive the change of rate of a clock with respect to TDB

$$\frac{d t_{CLOCK}}{d t_{TCB}} / \frac{d t_{TDB}}{d t_{TCB}} = \frac{d t_{CLOCK}}{d t_{TDB}} = \frac{\left[ 1 - \frac{2\varphi_{CLOCK}}{c^2} - \left( \frac{v_{CLOCK}}{c} \right)^2 \right]^{\frac{1}{2}}}{1 - L_b}. \quad (4)$$

The total difference in time between a S/C clock and TDB from a general relativistic point of view can be retrieved by integration of Equation (4) over time.

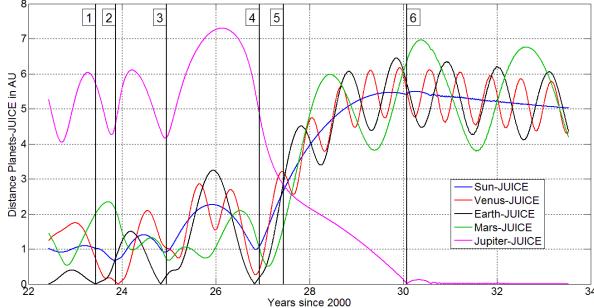


Figure 1: Distances of selected planets to the spacecraft throughout the mission

### 3. Application to the JUICE Mission S/C Clock

In preparation of the JUICE (Jupiter Icy Moons Explorer) mission we study the variations of the S/C clock with respect to TDB due to relativistic effects. Figure 1 shows the distance of the various planets with respect to the S/C throughout the mission with various mission events of the trajectory being highlighted with the numbers (1) to (6). After launch in June 2022 JUICE will fly to Jupiter on an interplanetary trajectory, including flybys at Earth in June 2023 (1), Venus in November 2023 (2) and again Earth in October 2024 (3) and October 2026 (4). After leaving the inner planets (5) JUICE will arrive in the Jupiter system around January 2030 (6). Two Europa and several Callisto and Ganymede flybys will follow while JUICE is in orbit around Jupiter. Finally JUICE will enter a Ganymede orbit where it remains until mission end in July 2033 [ 3 ]. Figure 2 shows the corresponding velocity of the S/C with respect to SSB throughout the mission. Large variations in velocity can be seen around the flybys, targeting the trajectory to the Jupiter system, as well as changing the trajectory in the Jupiter system Orbit Phase.

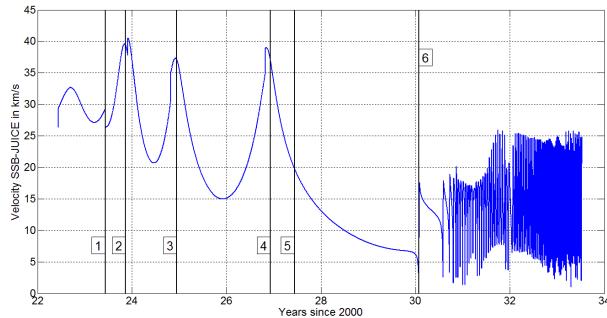


Figure 2: Velocity of S/C w.r.t. SSB

These changes occur on shorter time-scales than during the interplanetary cruise due to the orbits.

In order to retrieve the variation of the rate of the clock we used the NAIF SPICE toolkit in combination with JPL's planetary ephemerides DE430 and the JUICE Trajectory Kernel. For the calculation of the potential at the clock's location the Sun, Mercury, Venus, Earth and its Moon, Mars, Jupiter and its moons Io, Europa, Ganymede, Callisto were incorporated. Since TDB was chosen as the reference system the TDB values of their gravitational constants were taken from [ 4 ]. By using the distances of the incorporated bodies to the S/C at a certain point in time  $\varphi$  was calculated for every point along the trajectory.

Figure 3 shows the deviation of the S/C clock rate with respect to TDB plotted as  $1 - d t_{CLOCK} / d t_{TDB}$  in unit of  $10^{-8} \text{ s/s}$ . The Markers (1) to (6) were derived from this curve and applied to all the other plots, in order to identify the largest changes in the S/C clock rate.

The jump at (1) is related to the first Earth flyby, while during the peak at (2) the S/C performs a Venus flyby and reaches its closest distance to the sun of the mission. The peak at (3) is related to the second Earth flyby, as well as close approaches of Sun, Venus, Mars and Jupiter during their trajectory variations respectively. (4) marks the third and last Earth flyby while again the Sun, Venus and Mars are close at that time. (5) represents the point of transition from the inner planets to the Jupiter system along the S/C trajectory. From that point on the influence of Jupiter's gravitational potential is steadily increasing as the S/C is getting closer to it. (6) marks the arrival of the S/C in the Jupiter system. Owing to the short orbital periods of the orbits in the Jupiter System, changes of the rate are on shorter time scales. Most of the time the S/C clock runs faster than TDB, except around the events (2) and (3) – cf. Figure 3.

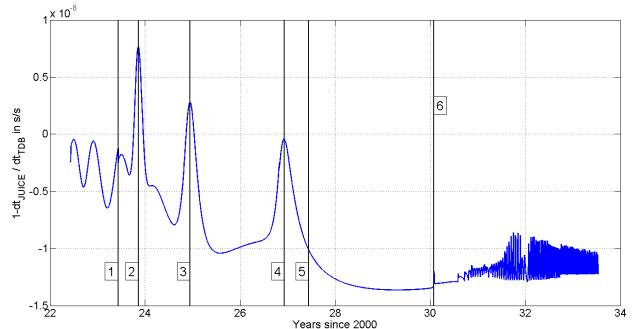


Figure 3: Deviation of S/C clock rate w.r.t. TDB

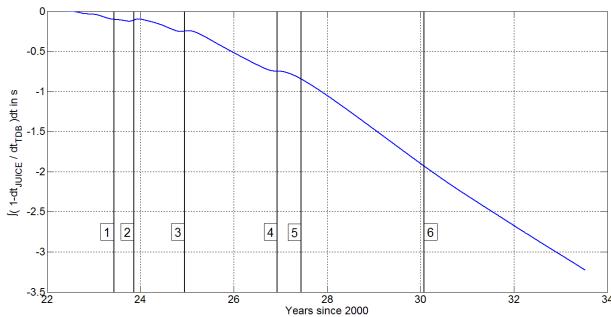


Figure 4: Time offset between S/C clock and TDB

Figure 4 shows the accumulated offset in time between the S/C clock and TDB, plotted as  $\int (1 - d t_{CLOCK} / d \tau_{TDB}) dt$ . At the end of the mission ( $\approx 11.5$  years) the S/C clock is  $\approx 3.22$  s ahead of TDB, clearly demonstrating the relevance of general relativistic effects for the mission.

## 4. Summary and Conclusions

Since timing is a crucial element for Space Missions, we concluded that effects influencing S/C clocks have to be taken into account. Within this study we investigated the influence of relativistic effects on S/C clocks, deriving a relationship for the difference in rate between the Dynamical Time TDB and the S/C clock. From that we can integrate the offset in total time and therefore providing a precise link between TDB and S/C On-board clock Time.

By using the trajectory Kernel of the JUICE Mission we studied the differences between TDB and the S/C clock. We identified the flybys during interplanetary cruise as the main reason for the strong variations of the S/C clock rate. During the Jupiter System Orbit Phase, the clock rate is steadily changing, due to the steadily changing orbit of the S/C and the gravitational interaction with Jupiter and its moons. After  $\approx 11.5$  years an offset of  $\approx 3.22$  s has been accumulated.

We consider Laser Ranging from Ground Stations (e.g. International Laser Ranging Service – ILRS) to the Ganymede Laser Altimeter (GALA) instrument on-board the JUICE S/C a perfect technique for providing precise tracking data. Such range measurements with a typical precision of 10 cm [5] would be suitable for the observation of the described effects and precise clock calibration.

## References

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