

HELIOS: A Radiative Transfer Solver for Exoplanets

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Abstract

Exoplanet atmospheres become a key laboratory for radiative transfer, which requires to be explored for a broader range of atmospheric condition than our Solar System atmospheres. In this study, we develop a package of codes that includes a k -distribution calculator, a radiative transfer solver for exoplanet atmospheres based on a numerical method, and a calculator for radiative equilibrium temperature in both optically thin regime (two-stream approximation) and optically thick regime (flux-limited diffusion approximation). Given a high-speed Graphics Processing Unit (GPU) platform, we take advantage of a parallel computing which allows an acceleration of radiative transfer calculation while tabulating k -distributions, integrating transmissions, etc. We present benchmark tests for the pure H₂O case, ranging from a line-by-line calculation for a set of a small number of H₂O lines to a complete temperature profile in radiative equilibrium for a heterogeneous type of exoplanet atmospheres. The purpose of the tests is to validate our model before we adopting our radiative transfer code to a 3-dimensional general circulation model (GCM).

1. Introduction

A radiative transfer solver for planetary atmospheres is a versatile tool that enables us to understand the theoretical aspects of thermal structure and chemistry of atmospheres in radiative equilibrium, to interpret the remotely-sensed spectra of transiting or directly-imaged exoplanets, and to calculate the heating and cooling rates for a given thermal condition in general circulation models (GCMs). The solution of radiative transfer in planetary atmospheres has long been discussed in the literature (e.g. [1, 2, 3]) and they provided insights to the radiative equilibrium states that can be numerically explained by an approximation, i.e., “two-stream approximation”. However, a clear description on the process that covers from molecular line to heating/cooling rate in a column of atmo-

sphere has not been clearly accomplished for exoplanets, e.g., hot Jupiters. In this sense, the main objective of this study is to introduce the benchmark cases for each step of radiative transfer calculation using a pure H₂O atmosphere, which are useful for the validation of radiative transfer codes available in the community. As a significant improvement over previous methods, we address a parallel computing technique so that we can accelerate a radiative transfer calculation by a factor of ~ 100 compared to a serial computing technique.

2. Methodology

We present the wavelength-dependent transmission calculation based upon a correlated k -distribution technique and a radiative transfer solver using both a two-stream approximation and a flux-limited diffusion method. We introduce a set of algorithms that all take advantage of a parallel computation accelerated by the GPU architecture.

2.1. k -distribution calculator

Calculating transmission for a given temperature and pressure in the present study is performed based on a correlated k technique [4, 5], where the mean transmission is defined by

$$\overline{T} \approx \sum_{i=1}^N e^{-k(\mathcal{G}_i)m} \Delta \mathcal{G}_i. \quad (1)$$

Here m is the abundance of molecule and N is the number of \mathcal{G} -abscissae, where $k(\mathcal{G}_i)$, i.e., k -coefficients, are calculated. A set of k -coefficients in a given wavelength bin is computed in a parallel computing framework and tabulated in a grid of temperature, pressure, and wavelength.

2.2. Two-stream approximation

In the purely absorbing limit, the upward and downward fluxes (F_{\uparrow} and F_{\downarrow}) at an interface between two

adjacent layers (Layer 1 and 2) can be computed by integrating over hemispheres, yielding

$$F_{\uparrow 1} = F_{\uparrow 2} \mathcal{T} + \pi B (1 - \mathcal{T}), \quad (2)$$

$$F_{\downarrow 2} = F_{\downarrow 1} \mathcal{T} + \pi B (1 - \mathcal{T}), \quad (3)$$

where \mathcal{T} is the transmission function of a layer that is calculated from k -coefficients in Section 2.1 and B is the Planck function. Here we assume isothermality in each layer, i.e., $\partial B / \partial \tau = 0$. An approximate way to write the transmission function is

$$\mathcal{T} \approx \exp(-\mathcal{D} \Delta \tau), \quad (4)$$

where \mathcal{D} is the diffusivity factor, which can be treated in various ways, and $\Delta \tau \equiv \tau_2 - \tau_1$. Two-stream approximation is accurate in optically thin atmospheres.

2.3. Radiative Equilibrium Temperature

For optically thin atmospheres, the net flux in a layer is the sum of the incoming fluxes from neighboring layers and the outgoing fluxes towards neighboring layers.

$$\Delta F = (F_{\uparrow b} + F_{\downarrow a}) - (F_{\downarrow b} + F_{\uparrow a}). \quad (5)$$

The subscript b and a mean that the departure of the flux is the layer below and above. The heating rate of a layer is now

$$\mathcal{H} = -\frac{1}{c_p \rho} \frac{\Delta F}{\Delta z} (K/day), \quad (6)$$

where c_p , ρ , and Δz are the specific heat capacity at constant pressure, mass density, and layer thickness, respectively. A new temperature is

$$T' = T + \mathcal{H} \Delta t, \quad (7)$$

where Δt is the time step in day, which determines the speed of radiative relaxation to radiative equilibrium. In radiative equilibrium, the flux change between neighboring layers are static with time step, i.e., $\Delta F / \Delta t = 0$ for all layers.

For optically thick atmospheres, two-stream approximation with isothermality is no longer valid ($\partial B / \partial \tau \neq 0$) if the vertical resolution of atmosphere is insufficient, where a flux-limited diffusion approximation is much more reasonable to describe radiative transfer. Then the temperature from diffusion is

$$T_{diff.} = T_{int} \left(\frac{2}{3} + \frac{3}{4} \frac{\kappa_R P}{g} \right)^{\frac{1}{4}}, \quad (8)$$

where T_{int} is the intrinsic temperature, which determines the internal heat condition of the atmosphere, and κ_R is the Rosseland mean opacity. P and g are pressure and gravity. Therefore, the final temperature profile is characterised by a combination of the flux contributions from T' and $T_{diff.}$.

3. Summary and Conclusions

We developed the GPU-accelerated algorithms for the description of radiative transfer in heterogeneous exoplanet atmospheres and presented a series of benchmark tests which can be useful to validate radiative transfer codes from the community. With a successful development, we will implement the codes into the 3-D GCM for exoplanets that are currently under development (see the presentation by Mendonca et al.).

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References

- [1] Chandrasekhar, S.: Radiative Transfer, Dover Publications, 1960
- [2] Meador, W. E. and Weaver, W. R.: Two-stream approximations to radiative transfer in planetary atmospheres: A unified description of existing methods and a new improvement, *Journal of the Atmospheric Sciences*, Vol. 37, pp. 630-643, 1980
- [3] Toon, O. B., McKay, C. P., Ackerman, T. P., and Santhanam, K.: Rapid calculation of radiative heating rates and photodissociation rates in inhomogeneous multiple scattering atmospheres, *Journal of Geophysical Research*, Vol. 94, pp. 16287-16301, 1989
- [4] Goody, R. M., and Yung, Y. L.: Atmospheric radiation: theoretical basis, Oxford University Press, 1989
- [5] Lacis, A. A., and Oinas, V.: A description of the correlated k distribution method for modeling nongray gaseous absorption, thermal emission, and multiple scattering in vertically inhomogeneous atmospheres, *Journal of Geophysical Research*, Vol. 96, pp. 9027-9063, 1991