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Interpretation of the free libration of the Moon for the observations

N.Petrova (1,2), A. Gusev (1), Yu. Nefediev (1), H. Hanada (3)
(1) Kazan Federal University, Kazan, Kremlevskya str., 18, Russia, (2) Kazan Power Engineering University, Kazan, Russia, (3) National Astronomical Observatory of Japan, Mizisawa, Japan (nk_petrova@mail.ru)

Abstract

Main results in investigation of the lunar physical libration in the Kazan University are presented in the report. Modern problems in the lunar spin-dynamics are considered. The accent is done on the fine phenomena of the lunar libration caused by complicated interior structure. Parameters of a free libration are discussed; geometrical interpretation of the chandler-like and free core nutation is given.

1. Solution for the Lunar pole's motion. «Free» and «arbitrary» libration of the Moon

We have constructed the Hamiltonian for free rotation of the two-layer Moon. However, the Eulerian motion of a body relative to the centre of mass that occurs of inertia, i.e in the absence of external forces, does not match the real lunar rotation, which is described in the first approximation by Cassini's laws. Physically it means that at absence of external forces the Moon is being not moved along an orbit and is not focused by one its part to the Earth. Therefore, to consider absolutely free rotation of the Moon is not meaningful. It's necessary, as a minimum, to describe the lunar rotation in the gravity field of the Earth, i.e. to consider the resonant character of lunar rotation, including into consideration the potential of interaction of the Lunar body with the Earth (and with the Sun, in a more precise case).

By constructing the potential for a two-layer Moon and by selecting of it the terms responsible for the resonant rotation of the Moon, described by Cassini's laws, we, following the Getino approach [2], have found solution of Hamilton equations for eigenfrequencies of the lunar rotation[1]:

$$\sigma_1 = \Omega \frac{A}{A_m}\sqrt{\kappa\frac{(C-A)(C-B)}{AB}} = \Omega\frac{A}{A_m}\sqrt{\kappa\alpha\beta} \quad \rightarrow \quad \text{CW}$$

$$\sigma_2 = -\Omega\left[1+\frac{A}{2A_m}\left(\frac{C_c-A_c}{A_c}+\frac{C_c-B_c}{B_c}\right)\right] = -\Omega\left[1+\frac{A}{2A_m}(e_{ca}+e_{cb})\right] \rightarrow \quad FCN$$

Unlike the Eulerian rotations, for the frequency σ_1 called usually as a Chandler-like wobble the factor $\kappa \approx 4$ appears under a square root. It comes into the solution from the potential. Such libration can not be called the "free" libration, because it occurs in the orbital motion of the Moon, i.e. under action of the external force. Hayn (1923) has received the harmonics with the arbitrary constants and gave explanation of the physical sense of the expressions received by him and consequently named them not a "free libration" (Freie Libration), but the "arbitrary libration" (Willkürliche Libration). As a result there is distinguish in the formula for the period of Chandler-like wobble with the Getino's results, obtained for the Eulerian rotation of the Earth.

The frequency σ_2 appears in the solution due to the core and represents so called Free Core Nutation (FCN). A sign "minus"before the expression denotes that oscillation occurs in retrograde direction relative to orbital motion. The rotation axis of the core and the mantle are not aligned, and then the FCN takes place. Displacement of the axes may be explained by various reasons and, one of them is the fact that the mantle is subjected to the Earth attraction, but the core is rotated freely due to its small mass.

To each of the received frequencies the respective period can be set.

$$P_{CW} = \frac{P_{rot}}{\sqrt{\kappa}\sqrt{\alpha\beta}}\frac{A_m}{A} \cong \frac{P_{rot}}{2\sqrt{\alpha\beta}}\frac{A_m}{A} = \frac{1}{2}P_{Eulerian} = 74.07\,years$$

Here $P_{rot} = 27.3$ days is the sideral period of Lunar rotation, α, β are ellipticity parameters of a lunar body, A_m, A are main moments of inertia for the mantle and whole Moon, respectively. The value of P_{CW} is twice less ($\sqrt{\kappa}$) than the Eeulerian period.

This fact most evidently shows distinction between the arbitrary and Eulerian polar motion.

For the second frequency $\sigma_2 = -\left[\frac{1}{1\text{month}} + \frac{1}{P_{FCN}}\right]$ the respective period P_2 is close to the Lunar month. It is caused by the existence of a lunar core and is called Free core nutation (FCN). The FCN-period for an three-axial core is expressed as $P_{FCN} = -\frac{P_{rot}}{2(e_{ca}+e_{cb})}\frac{A_m}{A}$.

Our estimations of the P_{FCN} = 144 years ÷186 years. Such great values (relative to the Earth and Mars) of Lunar free periods are explained by the small size of a core and by the slow Lunar rotation [3].

2. Geometrical and physical interpretation of the periods

As a result of solution of Hamiltonian equations the projections of angular velocity vector $\vec{\omega}$ on the principal axes were obtained. If on a lunar pole a telescope will be posed, as it's planned in Japanese project ILOM [], and the axes of field of view of the telescope will be oriented along the principal axes (Fig. 1), then obtained solution will describe the motion of the angular velocity vector due to the free libration.

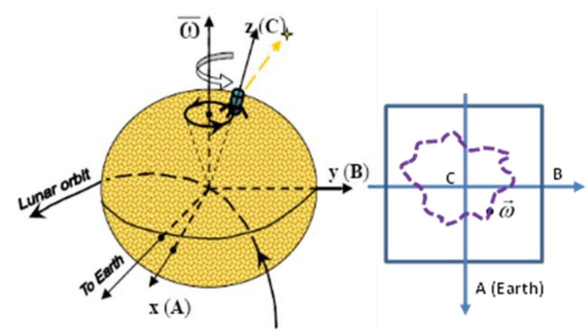

Figure 1: Motion of the angular velocity vector in the field of view of the lunar polar telescope.

This motion on a Lunar surface may be considered as consisting of two motions over ellipses (Fig. 2). The pole moves in the retrograde direction over a small ellipse with semi-axes F_1 and F_2 with a period equal approximately to Lunar month, its centre moves along the ellipse with semi-axes D_1 and D_2 in the prograde direction with the period of 74 years.

Amplitudes D1=3.31″, D2=8.19″ were determined from longterm (more than 40 years) laser location []. At the same time the FCN amplitudes should be too small. According to Yoder estimations [], amplitudes F_1 and F_2 are of a few milliseconds. Furthermore, because the FCN period is too close to the lunar month, it'll be difficult to detect this term in libration observations.

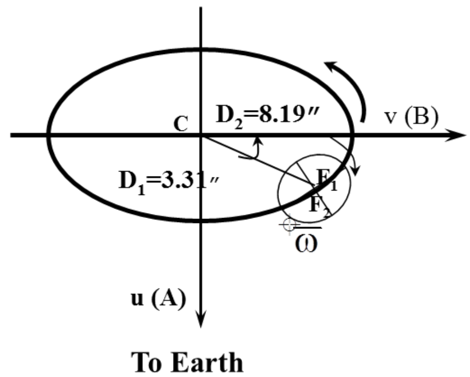

Figure 2: Motion of the angular velocity vector due to two kind of free libration.

3. Conclusions

Such fine effect as the FCN has been not observed from the Earth yet. Despite the difficulty to detect free libration understanding of how these types of free libration manifest is very important for realization of observations of lunar physical libration, including from the lunar surface.

References

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