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Simulating parameters of lunar physical libration on the basis of its analytical theory

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Abstract

Results of simulating behavior of lunar physical libration parameters are presented. Some features in the speed change of impulse variables are revealed: fast periodic changes in p_2 and long periodic changes in p_3. A problem of searching for a dynamic explanation of this phenomenon is put.

The simulation was performed on the basis of the analytical libration theory [1] in the programming environment VBA.

1. Analytical approach to the description of physical libration

Analytical theory of the physical libration of the Moon [1], built on the basis of solutions of Hamilton equation, presents the canonical variables - libration angles (q_1, q_2, q_3) (Fig. 1) and the conjugate impulses (p_1, p_2, p_3) - in the form of Poisson series. These series represent the dependence of canonical variables on time and on parameters of the gravitational field of the Moon. In this report we present dependence only on time of these variables. Study of the dependence on the parameters of the dynamic model of the Moon - Stokes coefficients - is given in detail in [2]. In this work, calculations were made based on the model LURE2 [3]. Of course, it is an older model compared to current models of LP or SGM, but in the present study, the numerical values of the gravitational field does not matter. We used the model LURE2, on which the theory [1] was built.

For practical applications, usually only the values of the angular variables are of interest, and the impulses play the role of auxiliary variables. However, the time variation of the impulse, as it turned out, has a number of features. We restricted ourselves to consideration of (q_2, q_3) and (p_2, p_3), as in the behavior characteristics of longitude no features were found.

2. Behavior of the canonical impulses

Fig. 1 shows the change in values of the direction cosines of the ecliptic pole q_2 and q_3. Apart the phase shift and the small difference in the amplitudes the both angles behave almost identically: the powerful amplitude is manifested in harmonic F, corresponding to 27.12 days, i.e. to Draconitic period. Other harmonics have amplitudes at least an order of magnitude smaller than the amplitude at sin / cos F, so the scale of the graph does not reflect these changes. Impulse p_2 behaves similarly to its conjugate variable q_2 (Fig. 3).

But the behavior of the impulse p_3, calculated at 5 months, is essentially different: for a period of 5 months we see only small periodic fluctuations in the background of a strong upward trend of the impulse p3. Calculation for a period of 1 year did not change the behavior impulse p_3.

3. Analysis of simulated diagrams

It is obvious that a systematic increase in the rate of change of the periodic variable q_3 should not be (the canonical impulse corresponds to the velocity of change of the conjugate canonical variable).

We estimate the analytic structure of the function p3 (Table 1) and found that the most powerful harmonic in the series for p_3 is the harmonic (l - F): its amplitude is two orders of magnitude larger than the amplitude other harmonics. Period of this harmonic is almost 6 years, so calculating the behavior of p_3 for this period, at Fig. 4 we saw the expected periodic behavior of the impulse.

This is a long-period component. It describes the so-called *elongation* - it's the angle between the mean direction of the lunar body "nose" (average x-axis at the equator) and the mean ecliptic direction to the center (the Earth), around which the Moon is moving.

As we understand, the changes in the rates of canonical variables are basically caused by balance of centripetal and centrifugal forces, that determine the resonant character of the spin-orbital motion of the moon. Geometric nature of q_2 shows that the main change in its velocity is due to the inclination of the dynamic equator to the ecliptic. Due to the growing difference between the x axis and the average direction of the Moon to the Earth, the rate of change of the angle q3 increases, but due to the tendency of the system to return to sustainable equilibrium state, this rate gradually changes its sign and decreases. Unfortunately we cannot give a more detailed dynamical explanation of this process. Owing to resonance the difference is small, and therefore the period of change is so long.

3. Conclusions

The analysis is useful for understanding the dynamic nature of changes in the libration angles FLL. Analytical theory, in this case, helps understand the reasons for many of the observed phenomena.

References

[1] *Petrova N.* (1996) Earth, Moon and Planets, **73**, 1, p. 71.

[2] *Petrova N., Hanada H.* (2012) Planetary and Space Science, Vol. 68. p. 86 – 93.

[3] *King, R. W., Counselman, C. C., Shapiro, J. J., and Williams, J. G.* (1975): LURE2, J. Geophys. Res. 81, 6251

Table 1: Fragment of the analytical series for p_3
Most significant harmonics are presented

	l	*F*	*l'*	*D*	*Amplitude* ($\times 10^5$)	*Period* (*day*)	*Period* (*year*)
1	0	0	0	0	9,01	const.term	const.term
-1	0	1	-1	-1	0,03	6792,3	18,6
-1	0	1	0	-2	-0,02	-32,3	-0,1
-1	0	1	0	-1	-0,01	346,6	0,9
-1	0	1	0	0	0,02	27,2	0,1
-1	1	-1	-1	0	-0,01	-313,1	-0,9
-1	1	-1	0	-2	0,02	-14,7	0,0
-1	1	-1	0	0	-13,80	-2190,3	-6,0
1	1	-1	0	0	0,01	-2190,3	-6,0
-1	1	-1	1	0	-0,01	438,4	1,2
-1	1	1	0	-2	0,28	188,2	0,5
-1	1	1	0	0	-0,09	13,7	0,0
-1	2	-1	0	0	0,01	27,9	0,1

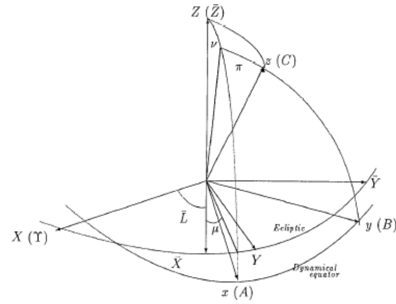

Figure 1: Description of the physical libration using the canonical variables $\mu = q_1$, $\nu = q_2$, $\pi = q_3$

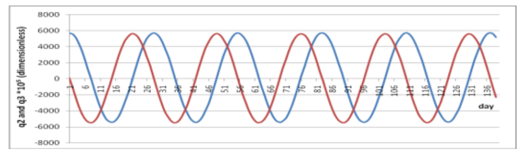

Figure 2: Behavior of angles q_2 (blue) and q_3 (red) at the time interval in 138 days.

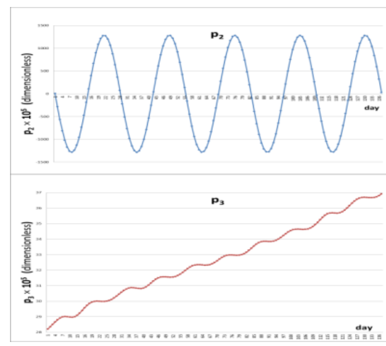

Figure 3: Behavior of impulses p_2 and p_3 at the time interval in 138 days.

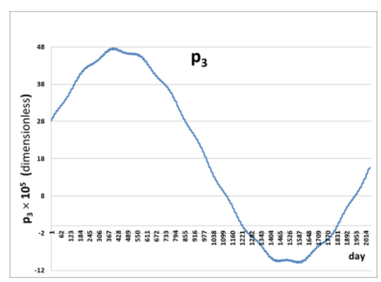

Figure 4: Behavior of impulses p_3 at the time interval in 5.9 years.