EPSC Abstracts
Vol. 9, EPSC2014-744, 2014
European Planetary Science Congress 2014


# Excess noise in synthetic stellar occultation data from N-body simulations of Saturn's rings

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## Abstract

The excess variance in stellar occultation measurements, as compared to that expected from Poisson statistics, provides an useful tool for extracting information of the ring particle size distribution [1] and/or the tendency of particles to form transient aggregates (e.g. [2]). We compare the excess variance calculated from N-body simulations with formulae derived in literature [1]. Besides highlighting some basic dependencies, we illustrate how the destruction of self-gravity wake structures at satellite density waves crests might manifest as a local reduction of effective particle size.


## 1. Introduction

The variance in the number of photons $k$ passing through a ring with a footprint area $A_d$ can be written as (Ref. [1]; hereafter SN1990)

$$\sigma^2(k) = E(k^2) - E^2(k) = \bar{\lambda} + {\lambda_s}^2 \sigma^2(P). \quad (1)$$

Here $\lambda_s$ is the number of original photons, $P$ is the probability for a photon to pass through the ring, and $\bar{\lambda} = \bar{P}\lambda_s$ is the expectation value of $k$. The measured optical depth $\tau = -\mu \log \bar{P}$, where $\mu = \sin B$, with $B$ denoting the illumination elevation. In case of a 'gray' uniform ring ($P$ =constant), the observed $k$ would follow Poisson statistics, $\sigma^2(k) = \bar{\lambda}$. However, in real rings $P$ is a random variable that varies between adjacent occultation footprints. The excess noise

$$\frac{\sigma^2(k) - \bar{\lambda}}{{\lambda_s}^2} = \sigma^2(P) \quad (2)$$

depends on the graininess of the underlaying particle distribution. This is affected both by the particle size distribution (effective size vs footprint size) and the spatial distribution (e.g. particle clustering due self-gravity wakes or other local clumpiness). SN1990 derived a formula for the excess variance due the particle size distribution (we ignore diffraction which is also accounted by SN1990).

$$\sigma^2(P) = e^{-2\tau/\mu}(e^{Q\tau/\mu} - 1). \quad (3)$$

Here $Q = \pi {R_{\text{eff}}}^2/(\mu A_d)$, with ${R_{\text{eff}}}^2 = \frac{<R^4>}{<R^2>}$, carries information of the size distribution. As noted by SN1990, the $R_{\text{eff}}$ derived with this formula is also sensitive to non-uniformities in particles distribution,

## 2. Simulation results

We have applied the SN1990 treatment to synthetic particle fields, and compared the measured excess variance with that derived from Eq. (3). Like in [3] we use both uniform particle fields produced by random placement of non-overlapping particles, and particle fields from N-body simulations of Saturn's rings. In these comparisons, the effective particle size is known, and the excess noise is measured using different numbers and sizes of footprints, and numbers of photons/footprint. For example, experiments with uniform systems with various volume filling factors $D$, optical depths, and particle size distributions confirm that Eq. (3) holds reasonably well at the limit $D \to 0$ (Fig. 1), provided that the particle size is not negligible compared to the footprint size. However, for non-gravitating N-body rings, which are also spatially uniform but have $D > 0$, the excess noise is smaller than predicted by Eq. (3) (Fig. 2). In this sense the high filling factor seems to reduce the randomness of the particle distribution.

If the system is not uniform but contains microstructure, the excess noise can become, as expected, much larger than predicted by SN1990 formula: Fig. 2 shows examples of self-gravitating simulations. In this case the noise is very sensitive to the strength of the self-gravity wakes and thus depends on factors like particle internal density and elasticity.

In Fig. 3 we illustrate how the simulated noise might behave in a density wave region. The simulation uses the modified local method introduced in [4] which takes into account the periodic compression/decompression as the simulated region enters/leaves the density crests. The evolution of the system is followed over 5 orbital periods. Initially, the $R_{\rm eff}$ calculated via Eq. (3) increases while the system develops self-gravity wake structures. During the subsequent evolution the $R_{\rm eff}$ seems to be smallest when the system enters the density crests. Most likely this is due to temporary destruction of the wake structure, reducing the excess variance. In Ref. [2] such a reduction in $R_{\rm eff}$ has been observed in high-resolution UVIS occultations of density wave regions.

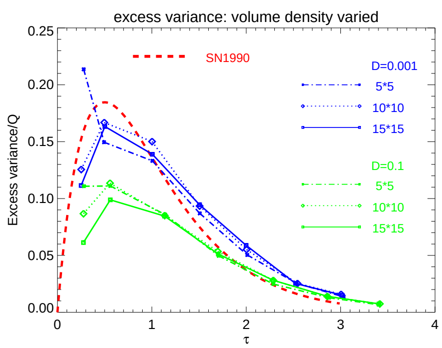


Figure 1: Excess variance (normalized with Q) in simulations with randomly distributed non-overlapping particles. Two different volume filling factors $D$ are compared, with 3 different footprint sizes (relative to particle radius). Red dashed curve indicates the SN1990 theoretical curve.

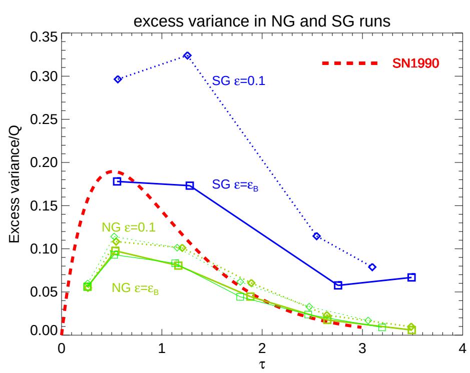


Figure 2: Comparison of N-body simulations with (SG) and without (NG) self-gravity. Impacts were modeled either with velocity dependent coefficient of restitution (Bridges $\epsilon_B$) or with a constant $\epsilon = 0.1$. The thin curves denote NG runs with different size distributions

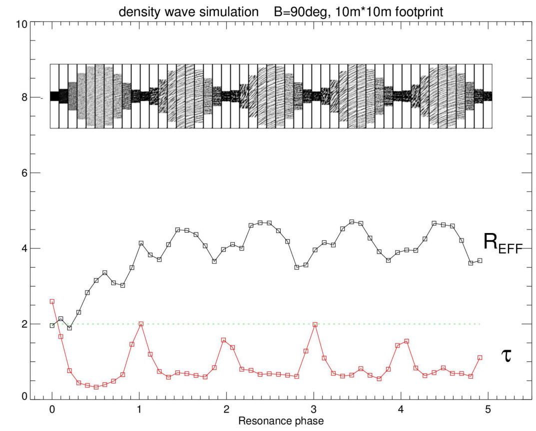


Figure 3: Simulation of the *local* response to a satellite density wave: at the density crests (peaks of optical depth) the effective particle size estimated with SN1990 formula is reduced, due to temporary destruction of self-gravity wakes. The dashed horizontal line indicates the true $R_{\rm eff}$; the increase in the apparent particle size depends on the assumed footprint size. The insert shows the time evolution of the simulation system (10 snapshots/period). In each snapshot the radial coordinate is upward: note the periodic variations in the self-gravity wake pitch angle and wavelength.

## Acknowledgments

This study is supported by the Academy of Finland.

## References

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