# Relativistic effects for the JUICE on-board clock

**S. Bauer (1)**, H. Hussmann (1), J. Müller (2), J. Oberst (1,3)

(1) German Aerospace Center (DLR) Berlin Germany, (2) Leibniz Universität Hannover, Germany (3) Technical University Berlin, Germany. Contact: sven.bauer@dlr.de

#### **Abstract:**

We studied relativistic effects on spacecraft clock rates for ESA's JUICE mission and derived a relationship between dynamical time TDB and on-board time. We analysed the S/C on-board clock rate by using the JUICE mission nominal trajectory. We identify significant changes in the rate of the clock due to large changes of the S/C velocity and its distance to the solar system bodies during the various spacecraft operational phases. After  $\approx 11.5$  years, at the end of the mission, the offset in time between dynamical time TDB and the on-board clock may run up to  $\approx 3.22$  seconds due to relativistic effects.

#### 1. Introduction

Accurate timing is critical for deep space missions since events have to be tagged correctly or commands have to be executed at the right time, e.g., for labelling when a certain measurement was taken or when a maneuver has to be executed.

For certain applications precise timing is required, as for example for range measurements from ground stations to the S/C. Since events happening at different locations in the Solar System have to be related with each other, the formulation of the equations and the included effects has to be quite precise.

#### 2. Influence of relativistic effects on S/C Clocks

Clocks are the crucial elements for measuring the timing and are influenced by various effects, which change their rate and therefore the timing itself. Besides stress e.g. from acceleration or vibration, change in pressure and humidity, hardware aging, electric or magnetic fields, radiation and temperature [1] variations due to relativistic effects can be observed. Changes in the rate of a clock occur due to changes of the potential at, and velocity of the clock. Following Moyer [2] one can describe the change of the rate of a clock by

$$\frac{d t_{CLOCK}}{d t_{TCB}} = \left[1 - \frac{2\varphi}{c^2} - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} \tag{1}$$

with arphi being defined as

$$\varphi = \sum \frac{\mu_i}{r_i} = \sum \frac{G \cdot m_i}{r_i}.$$
 (2)

Thereby TCB (Barycentric Coordinate Time) represents a coordinate time measured with an atomic clock in an inertial frame located at the SSB (Solar System Barycenter).  $\varphi$  is the potential at the clock's location, summed up over all bodies that are taken into account with their gravitational parameter  $\mu_i = G \cdot m_i$ . Thereby  $r_i$  is the distance between the clock and a certain body. While c is the speed of light, v represents the velocity of the clock with respect to the SSB.

The dynamical time TDB (Barycentric Dynamical Time) is the standard for planetary ephemerides and with respect to TCB defined as

$$\frac{d t_{TDB}}{d t_{TCB}} = 1 - L_b = 1 - 1.550519768 \cdot 10^{-8}.$$
 (3)

By combining Equation (1), (2) and (3), we can derive the change of rate of a clock with respect to TDB

$$\frac{d t_{CLOCK}}{d t_{TCB}} / \frac{d t_{TDB}}{d t_{TCB}} = \frac{d t_{CLOCK}}{d t_{TDB}}$$

$$\frac{d t_{CLOCK}}{d t_{TDB}} = \frac{\left[1 - \frac{2\varphi_{CLOCK}}{c^2} - \left(\frac{v_{CLOCK}}{c}\right)^2\right]^{\frac{1}{2}}}{1 - L_b}.$$
(4)

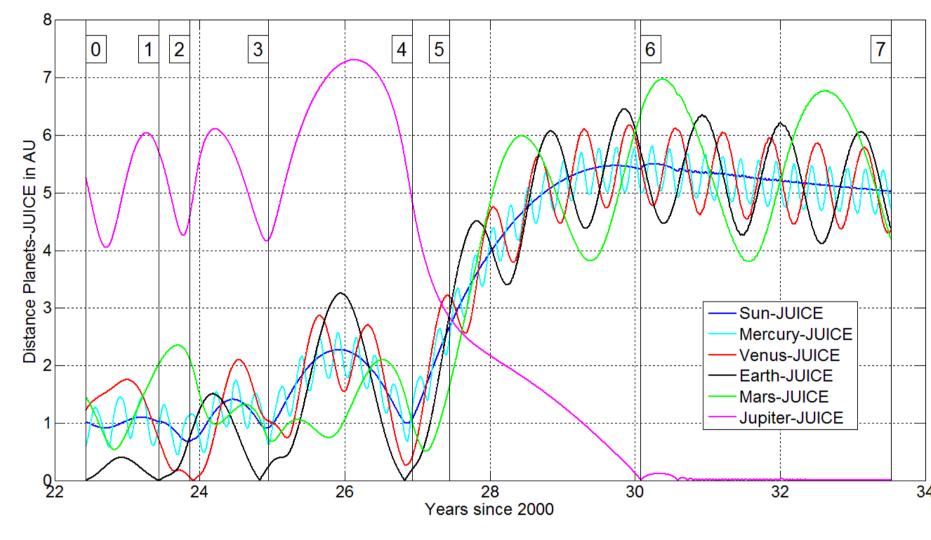
The total difference in time between a S/C clock and TDB from a general relativistic point of view can be retrieved by integration of Equation (4) over time.

### 3. Application to the JUICE mission S/C clock

In preparation of ESA's JUICE (Jupiter Icy Moons Explorer) mission we studied the variations of the S/C clock with respect to TDB due to relativistic effects. Figure 1 shows the distances of the planets taken into account with respect to the S/C throughout the mission. Selected mission events of the trajectory are highlighted with the numbers (0) to (7). After launch in June 2022 (0) JUICE will fly to Jupiter on an interplanetary trajectory, including flybys at Earth in June 2023 (1), Venus in November 2023 (2) and again Earth in October 2024 (3) and October 2026 (4). After leaving the inner planets (5) JUICE will arrive in the Jupiter system around January 2030 (6). Two Europa, several Callisto and Ganymede flybys will follow while JUICE is in orbit around Jupiter. Finally JUICE will enter a Ganymede orbit where it remains until mission end in July 2033 (7) [3]. Figure 2 shows the absolute velocity of the S/C with respect to the SSB throughout the mission. Large variations in velocity can be seen around the flybys, targeting the trajectory to the Jupiter system and during the orbit phase in the Jupiter system. These changes occur on shorter time-scales than during the interplanetary cruise due to the orbits around Jupiter and its moons.

In order to retrieve the variation of the rate of the clock we used the NAIF SPICE toolkit in combination with JPL's planetary ephemerides DE430 and the JUICE nominal mission trajectory kernel. For the calculation of the potential at the clock's location the Sun, Mercury, Venus, Earth and its Moon, Mars, Jupiter and its moons Io, Europa, Ganymede, Callisto were incorporated. Saturn was not considered in the analysis because its minimum distance to JUICE throughout the mission is ≈ 8.3 AU. Since TDB was chosen as the reference system, the TDB values of the gravitational parameters were taken from [4]. By using the distances of the incorporated bodies to the S/C at a certain point in time  $\varphi$  was calculated for every point along the trajectory. Figure 3 shows the deviation of the S/C clock rate with respect to TDB plotted as  $1 - d t_{CLOCK}/d t_{TDB}$  in units of  $10^{-8} s/s$ . The Markers (1) to (6) were derived from this curve and applied to all the other plots, in order to identify the reasons for the largest changes in the S/C clock rate. The jump at (1) is related to the first Earth flyby, while during the peak at (2) the S/C performs a Venus flyby and reaches its closest distance to the Sun throughout the mission. The peak at (3) is related to the second Earth flyby, as well as close approaches of Sun, Venus, Mars and Jupiter during their trajectory variations respectively cf. 1. (4) marks the third and last Earth flyby while again the Sun, Venus and Mars are close at that time - cf. 1. (5) is interpreted as the point of transition from the inner planets to the Jupiter system along the S/C trajectory. From that point on the influence of Jupiter's gravitational potential is steadily increasing as the S/C is getting closer to it. (6) marks the arrival of the S/C in the Jupiter system. Owing to the short periods of the orbits in the Jupiter System, changes of the rate are on shorter time scales.

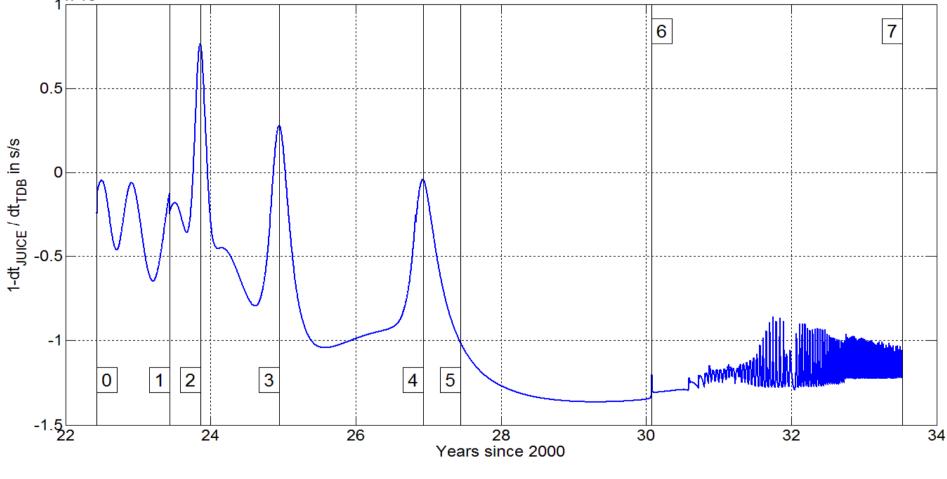
Most of the time throughout the mission the S/C clock runs faster than TDB, except around the events (2) and (3) – cf. 3. Figure 4 shows the accumulated offset in time between the S/C clock and TDB, plotted as  $\int (1 - d t_{CLOCK}/d \tau_{TDB})dt$ . At the end of the mission after  $\approx$  11.5 years, the S/C clock is  $\approx$  3.22 s ahead of TDB, clearly demonstrating the relevance of general relativistic effects for the mission.



40 35 30 20 20 20 10 0 1 2 3 4 5 0 2 24 26 28 30 30 32 3

Figure 1: Distances of selected planets to the S/C throughout the mission

Figure 2: Absolute velocity of the S/C w.r.t. SSB



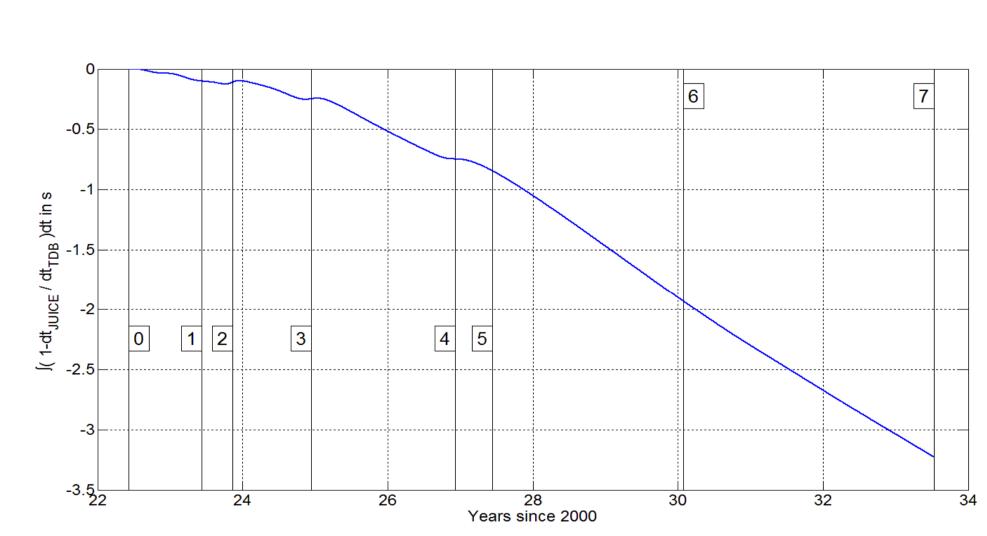


Figure 3: Deviation of the S/C clock rate w.r.t. TDB

Figure 4: Time offset between the S/C clock and TDB

# 4. Summary and conclusions

Since timing is a crucial element for deep space missions, we concluded that effects influencing S/C clocks have to be taken into account. Within this study we investigated the influence of relativistic effects on S/C clocks, deriving a relationship for the difference in rate between the dynamical time TDB and the JUICE S/C clock. From that we can integrate the offset in total time and therefore provide a precise link between TDB and S/C on-board clock time. By using the nominal mission trajectory of the JUICE S/C we studied the differences between TDB and the S/C clock. We identified the flybys during interplanetary cruise as the main reason for the strong variations of the S/C clock rate. During the Jupiter System orbit phase, the clock rate is steadily changing, due to the steadily changing orbital periods of the S/C and the gravitational interaction with Jupiter and its moons. After ≈ 11.5 years an offset of ≈ 3.22 s has been accumulated.

We consider Laser Ranging from Ground Stations (e.g. ILRS - International Laser Ranging Service) to the GALA (Ganymede Laser Altimeter) instrument on-board the JUICE S/C a perfect technique for observing the described effects and providing precise tracking data at the same time. Such range measurements show a typical precision of 10 cm [ 5 ] and would therefore be suitable for performing precise clock calibration and S/C positioning.

## References:

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