

About the vertical gradient of composition in Titan's lakes

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Abstract

The hydrocarbons seas of Titan, discovered by Cassini/Huygens mission are among the most interesting features of this object. However, their chemical composition remains not well known. Due to the presence of the methane in the atmosphere, only a few indications favoring the existence of some amount of ethane in Ontario Lacus have been brought by observations reported in [2, 10]. Several numerical models have been proposed: Dubouloz *et al.* (1989), Cordier *et al.* (2009,2013) based on the Regular Solution Theory, Glein *et al.* (2013) [4] and Tan *et al.* (2013) [13] respectively based on a RST family model and on the advanced equation of state PC-SAFT¹ [5, 13, 10, 3]. The atmosphere of Titan is dominated by nitrogen and contains a few percents of methane. The latter, photolyzed by solar radiations in the stratosphere, gives rise to a complex organic chemistry yielding to the production of a plethora of compounds [7]. According to numerical models, the most abundant species, produced by photochemistry, should be ethane. Then, the bulk composition of Titan's lakes can reasonably regarded as a mixture of methane and ethane, with some amount of dissolved N₂. This latter has a melting temperature (63.3 K) much below than that for methane (around 91 K) and ethane (101 K determined by Streng, 1971; 89.2 K measured by Timmermans, 1935); as the ground temperature of Titan in the range 90–95 K, the nitrogen may play a role of an antifreezing solute.

In this work, we investigate the possibility of a vertical chemical composition gradient produced by the joint effects of the pressure and temperature gradients. For that purpose, we built an original model based on the hydrostatic equilibrium

$$\frac{\partial P}{\partial z} = -\rho g \quad (1)$$

where P is the pressure, ρ the density and g the gravity. A thermodynamic equilibrium is hypothesized

¹Perturbed-Chain Statistical Associating Fluid Theory

through the liquid

$$\frac{\partial \Phi_i x_i}{\partial z} = 0 \quad (2)$$

where Φ_i is the activity coefficient of the species i and x_i is its model fraction at the depth z . The coefficient Φ_i and the density ρ are computed in the frame of PC-SAFT.

Beside this, the vertical energy transport is assumed to be governed by the equation (3), adapted from [14, 15] who used the formalism originally introduced in [1]

$$\frac{\partial}{\partial z} \left((\lambda_{L,\text{mix}} + \lambda_e) \frac{\partial T}{\partial z} \right) + \frac{\partial Q}{\partial z} = 0 \quad (3)$$

$\lambda_{L,\text{mix}}$ is the molecular thermal conductivity of the liquid mixture (W.m⁻¹.K⁻¹) and λ_e represents the contribution to heat transport by the eddy diffusion, which coefficient κ_e scales with the wind speed over the lake and is given by

$$\kappa_e = \frac{0.0325 \exp(-0.01278z) u_r / 10}{1 + \beta \frac{\partial p}{\partial z}} \quad (4)$$

u_r is the wind speed, λ_e , and κ_e are simply linked by $\lambda_e = \rho C_P \kappa_e$. The Eq. (4) is also used by [15] who adapted it from [16] and [8]. The former determined the value of β and obtained $5 \times 10^5 \text{ m}^4 \cdot \text{kg}^{-1}$. In Eq. (3), the source term $\partial Q / \partial z$ is due to the absorption of sunlight governed by

$$Q(z) = (1 - \alpha) S_g \exp(-\eta(h - z)) \quad (5)$$

in which S_g is the light initial flux, η is the absorption coefficient and h stands for the liquid height. For individual compounds, several empirical estimation techniques are available. We choose the method developed by Latini and his co-workers, published in a series of papers and summarized in [12] (see p. 10.44). In that approach, the thermal conductivity $\lambda_{L,i}$ (W.m⁻¹.K⁻¹) of a compound i is given by

$$\lambda_{L,i} = \frac{A_i (1 - T_{r,i})^{0.38}}{T_{r,i}^{1/6}} \quad (6)$$

where

$$A_i = \frac{A^* T_{b,i}^\alpha}{M_i^\beta T_{c,i}^\gamma} \quad (7)$$

the parameters A^* , α , β , and γ , are shown in Table 10-4 of [12]. The current temperature is noted T while $T_{b,i}$ and $T_{c,i}$ are respectively the temperature at the boiling point and the critical temperature. The molecular weight (expressed in $\text{g}\cdot\text{mol}^{-1}$) is represented by M_i , the ratio $T_{r,i} = T/T_{c,i}$ is the reduced temperature. Concerning the thermal conductivity $\lambda_{L,\text{mix}}$ of liquid mixture, we adopted the rule from Vredeveld (1973) [12].

Since $\lambda_{L,\text{mix}}$ depends on the local temperature T , the differential equation (3) is non-linear, then the Crank-Nicholson scheme used by [15] is no longer applicable. Rather, we have employed a finite-differences implicit scheme.

Unfortunately, the methane triple point depression estimation, performed by Mitri *et al.* (2007) [11], relies on a misuse of Landau & Lifshitz (1969) [6]. In addition, this kind of calculation in the context of a triple point, is not relevant for a possible freezing in deep sea liquid, because it assumes the presence of a vapor phase in contact with the liquid.

Alternatively, in order to assess if methane or ethane freezes, we propose a calculation in which we compare the chemical potentials of a species in the liquid phase and in the corresponding solid phase. Then, when the vertical structure is obtained by solving the above mentioned equations, we check if the equation

$$\ln \Phi_i x_i = -\frac{\Delta H_{i,m}}{RT_{i,m}} \left(\frac{T_{i,m}}{T} - 1 \right) \quad (8)$$

is satisfied at some depth z in the sea. In this equation $\Delta H_{i,m}$ and $T_{i,m}$ denote respectively the melting enthalpy and the temperature of melting of the considered compound (*i.e.* methane or ethane), while Φ_i and x_i are provided by the previously computed vertical structure.

Our study is particularly relevant in the context of the possible future space missions. Particularly to ones that consists in *in situ* exploration of Titan's lakes (see for instance Lorenz *et al.* (2015) [9]).

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