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Bayesian statistical approach to binary asteroid orbit determination

I.D. Kovalenko (1), R.S. Stoica (2), D. Hestroffer (1), A. Doressoundiram (1)
(1) Paris Observatory, France, (2) Lille 1 University, France (ikovalenko@imcce.fr)

Abstract

Orbit determination from observations is one of the classical problems in celestial mechanics. Here we present a statistical approach to banary asteroids orbit determination based on the algorithm of Monte Carlo Markov Chain (MCMC). Furthermore, the present method can be used on the orbit determination in the Gaia mission program for the observations of binary asteroids.

1. Introduction

The study of binary asteroids is of particular interest because the process of their formation and evolution have began since the formation of the Solar System. Therefore, the detection of large number of them and their study may have important implications for understanding and verifications of theoretical models of dynamical evolution of the Solar System. the determined orbit allows to derive the mass, and consequently the density if the size is known, which are essential physical characteristics of objects.
Calculate the orbit of a single asteroid with great accuracy is much easier than the orbit of binaries and this problem is complicated by relative motion of its components. Orbit determination is an inverse problem: we determine orbital parameters from observations.

2. Orbital parameters and observations

Every complete observation of a visual binary asteroid supplies with three data: the time of observation t, the position angle θ of the secondary asteroid with respect to the primary, and the angular distance ρ between the two asteroids. The given coordinates are measured in the tangent plane which changes for each observation. We introduce a coordinate frame related to the tangent plane, where x-axis is directed to the North, the y-axis to the Est and the third z-axis is normal to the tangent plane and directed towards observer, and we transform visually observed coordinates of secondary asteroid with primary asteroid in the origin to rectangular coordinates x, y:

$$\begin{aligned} x &= \Delta\delta = \rho\cos\theta \\ y &= \Delta\alpha\cos\delta = \rho\sin\theta \end{aligned} \tag{1}$$

The elements which define the relative orbit are the six Keplerian elements $(a, e, i, \Omega, \omega, T)$ and the period of revolution P. The semi-major axis a and eccentricity e define the form and the size of the orbit's ellipse, the inclination i, the longitude of the ascending node Ω and the argument of periapsis ω define the the position of orbit plane relatively to the equatorial frame, the time of periapsis passage and the period define the relative position of components at specific time.

3. Statistical inversion problem

The statistical ranging method for simple asteroids has been investigated by J.Virtanen et al. [2] and was applied for binary asteroids relative orbit determination by D.Oszkievicz et al. [1]. The main distinction of our method consists in the choice of proposal orbits: on each iteration we vary directly the orbital parameters to fit a new orbit, while in the previous methods proposal orbits derived through the sampling of observational coordinates. Moreover, compared to [1] algorithm, we combine MCMC method with the simulated annealing for the global optimization problem in order to find the best solution.

3.1. Bayesian statistical approach

The input data are made of N observations at times $t = (t_1, ..., t_N)$, which are related with theoretical positions by the observational equation:

$$\varphi = \psi(S) + \varepsilon \tag{2}$$

$$\varphi = (x_1, y_1; ...; x_N, y_N) \tag{3}$$

$$S = (a, e, i, \Omega, \omega, T, P) \tag{4}$$

$$\varepsilon = (\varepsilon_{\rho 1}, \varepsilon_{\theta 1}; ...; \varepsilon_{\rho N}, \varepsilon_{\theta N}) \quad (5)$$

where φ is a set of N observations coordinates on the tangent plane. $\psi(S)$ is a computed sky-plane positions, S the orbit determination parameters, ε contains the observational errors.

The problem leads to determination of the conditional probability density of the elements S with given observations φ, that is the *a posteriori* probability. Using the Bayes theorem and assuming that the probability density of observations $p(\varphi)$ is invariant for any orbits , the *a posteriori* probability density of the elements is related to the *a priori* probability and the likelihood probability:

$$p(S|\varphi) \propto p(S)p(\varphi|S) \quad (6)$$

3.2. MCMC method

We use the Metropolis–Hastings algorithm with the simulated annealing in order to obtain a sequence of orbits through sampling parameters S. Thus, for each iteration t randomly generate a candidate set of parameters S' from a proposed distribution $Q(S' \rightarrow S_{t-1})$. Namely, we introduce uniform random deviates with a small variation of each parameter around the last accepted sampling. The acceptance criteria:

$$\alpha = \min \left\{ 1, \left[\frac{p(S'|\varphi)}{p(S_t|\varphi)} \right]^{1/T} \frac{Q(S_t \rightarrow S')}{Q(S' \rightarrow S_t)} \right\} \quad (7)$$

where $T = h(T)$, called the temperature, is a global time-varying parameter. Each new orbit is accepted $S_t = S'$ with probability α. The acceptance ratio α indicates how probable the new proposed orbit is, according to the distribution $p(S|\varphi)$. Thus, we will tend to stay in high-density regions of $p(S|\varphi)$.

4. Summary and Conclusions

The MCMC method in form of the Metropolis–Hastings algorithm, adding a globally convergent coefficient, allows to derive one orbit with the biggest probability density of orbital elements. Additionally, the sequence of possible orbits derives through the sampling of each orbital parameter determines the phase space of every possible orbit considering each parameter. Thus the proposed method can be used for the preliminary orbit determination with relatively small number of observations, or for adjustment of orbit previously determined. The Gaia mission will provide positional measurements with high accuracy, which we will use for orbit determination. Asteroid binaries with large ($\approx$ 100 km) and small ($\leq$ 10 km) primary bodies will be observed in the course of the Gaia program. For those objects improved orbits, and consequently masses can be computed using our proposed method.

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References

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