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A numerical method for true polar wander of a laterally heterogeneous planet

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Abstract

The displacement of a celestial body's rotational axis with respect to its surface feature, or true polar wander (TPW) is studied in this paper and a numerical method is established which can deal with laterally heterogeneous models. This method is validated by comparing the numerical results with the analytical results which are developed based on normal mode theory. The results show good agreement. A further study of the TPW on Mars with a model which contains varying mantle viscosity is being conducted with the established numerical method.

1. Introduction

Analytically, the dynamics of polar wander is governed by two equations. Firstly, Liouville equation gives the general dynamics of a rotational body. When no external torque is applied, it reads $\frac{d}{dt}(\boldsymbol{I}\cdot\boldsymbol{\omega})+\boldsymbol{\omega}\times\boldsymbol{I}\cdot\boldsymbol{\omega}=0$, where $\boldsymbol{I}$ is the inertia tensor and $\boldsymbol{\omega}$ is the angular velocity vector. Both values are defined in a body fixed coordinate system. The analytical approach requires another equation which describes the moment of inertia $\boldsymbol{I}$. As the moment of inertia is perturbed by a geophysical process,the mass within the body redistributes and as the rotation axis changes, so the altered centrifugal force also deforms the rotational body. The total moment of Inertia attributable to such process is given by [1]

$$\begin{aligned}\boldsymbol{I}_{ij}(t) = I\delta_{ij} + \frac{k^T(t)a^5}{3G} * [\omega_i(t)\omega_j(t) - \frac{1}{3}\omega^2(t)] \\ +[\delta(t) + k^L(t)] * C_{ij}(t)\end{aligned} \quad (1)$$

Where I is the moment of inertia of the homogeneous spherical body, G is the gravitational constant. $k^T(t)$ and $k^L(t)$ are the degree 2 tidal love number and load love number respectively. C_{ij} represents the change in the moment of inertia without considering the dynamic deformation and it is this value that triggers the polar wander. The second and third term in Equation 1 stands for the changes which derive from the perturbed centrifugal force and from the mass redistribution induced by the triggering load respectively.

The analytical approach contains two major restrictions: First, the love numbers $k^T(t)$ and $k^L(t)$ can generally only be obtained for a homogeneous model. Secondly certain assumptions which simplify Equation 1 in the frequency domain are required so that it can be analytically solved together with the Liouville equation. However, these assumptions may not be true for other celestial bodies other than Earth. As a result, it is necessary to seek a numerical approach with which a general laterally heterogeneous planet can be studied.

2 Methodology

2.1 Numerical solutions of Liouville equation

First, we show that with the information about change in the moment of inertia, Liouville equation can be solved numerically with iterations. For a small enough time step, we assume that the change of moment of inertia varies linearly and by linear theory, Liouville equation leads to

$$m_1(t) = \frac{\Delta I_{13}(t)}{C-A} + \frac{C\Delta\ \dot{I}_{23}\ (t)}{\Omega(C-A)(C-B)} \quad (2a)$$

$$m_2(t) = \frac{\Delta I_{23}(t)}{C-B} + \frac{C\Delta\ \dot{I}_{13}\ (t)}{\Omega(C-A)(C-B)} \quad (2b)$$

with angular velocity defined as $\boldsymbol{\omega} = \Omega(m_1, m_2, 1 + m_3)$. In each step, the polar wander $m_i(t)$ is given an initial estimate and then the correspondent change of moment of inertia is computed from Equation 1. This value is then fed into Equation 2 to obtain the new $m_i(t)$ and the iteration continues until the result converges. The comparison between this numerical method and the analytical one from [3] is shown in Figure 1.

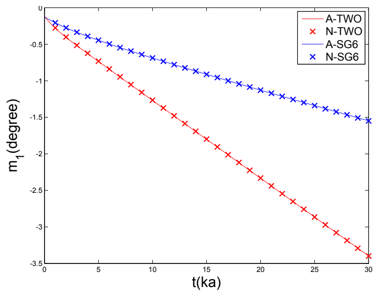

Figure 1: The polar wander path of two Earth models triggered by a point mass of 2×10^{19}kg placed at $45°$ colatitude in x-z plane. Lines shows the analytical(A) results and symbols represents the numerical (N) ones.

2.2 Finite element approach for change of moment of inertia

Next, we show that the change of moment of inertia can be numerically calculated instead of using Equation 1. [2] provides a finite element(FE) solution for calculating gravitationally self-consistent layered model by coupling the gravity term into the rheology equation through iterations. With information in the radius deformation, the change of moment of inertia for each layer can be calculated from

$$\Delta I_{ij,p} \simeq \int_S (\rho_{i+1} - \rho_i)(r_k r_k \delta_{ij} - r_i r_j) u_r dS \quad (3)$$

where ρ_i are densities of different layers and u_r is the radius displacement. When polar wander history is given and only the centrifugal force is considered for the laterally homogeneous model, the comparison between the analytical and FE results for calculating the change in moment of inertia is given in Figure 2.

For the theoretical non-zero components I_{11}, I_{22}, I_{33}, and I_{13}. The numerical result shows good correspondence with the analytical result. Theoretically, I_{12} and I_{23} should be zero. The results obtained from numerical methods have magnitudes which are about 4th order lower than the other four. These values represent the numerical errors which are estimated be around 0.1 %.

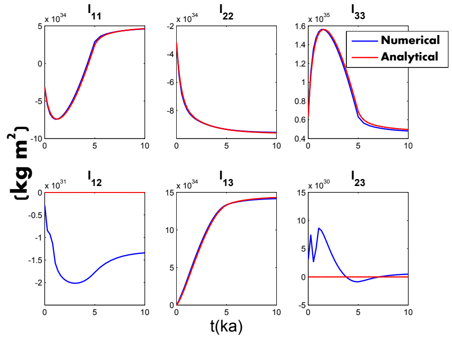

Figure 2: Change in moment of inertia for a two-layer Earth model with rotation axis linearly drifting from $0°$ to $45°$ within $x - z$ plane in 5 thousand years. The model is initially an unloaded sphere.

3 Conclusion

The governing equations for the polar wander are solved numerically and the change in the moment of inertia which is analytically calculated by Equation 1 can be obtained directly from a finite element model. Together, a numerical approach which can deal with laterally heterogeneous planet model is developed.

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