

# Statistical analysis of the Martian surface

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## Abstract

We investigate the scaling properties of the topography of Mars [10]. Planetary topographic fields are well known to exhibit (mono)fractal behavior. Indeed, fractal formalism is efficient to reproduce the variability observed in topography. Still, a single fractal dimension is not enough to explain the huge variability and intermittency. Previous study have shown that fractal dimensions might be different from a region to another, excluding a general description at the planetary scale. In this project, we are analyzing the Martian topographic data with a multifractal formalism to study the scaling intermittency. In the multifractal paradigm, the local variation of the fractal dimension is interpreted as a statistical property of multifractal fields. The results suggest a multifractal behaviour from planetary scale down to 10 km. From 10 km to 600 m, the topography seems to be simple monofractal. This transition indicates a significant in the geological processes governing the Red Planet's surface.

## 1. Introduction

The acquisition of altimetric data from Mars Orbiter Laser altimeter (MOLA) has motivated numerous analysis of the martian topography, in particular the surface roughness. A possible approach is to assume that topography can be mathematically described as a statistical field with quantitative parameters able to characterize the geologic units. Many statistical indicators have been proposed and widely explored in order to study the surface of Mars: RMS height, RMS slope, median slope [1], autocorrelation length [2]. Usefull informations have been obtained by the use of those indicators but they have the disadvantage of been defined at a given scale. By construction, they do not directly take into account the well-established scale symmetry that generally occurs in the case of natural surfaces. Indeed,

topography can not be interpreted as a stationary field, meaning that statistical parameters like the mean or the standard deviation exhibit a dependence toward scales. Hence the nature of this dependence needs to be accurately explained, otherwise the description of the surface remain incomplete. This subject has been widely studied in the past, parallel to the development of the notion of fractals ([3]). It is now well established that topography is efficiently modelled by fractal simulations. More interestingly, the fractal theory provides a mathematical formalism to describe the scale dependence of statistical parameters toward scales. It turns out that simple power-law relations efficiently approach the variability of planetary surfaces. The associated power-law exponent provides a quantitative parameter that is a good scale-independent candidate to characterize the geometric properties of a natural surface. A common example is given by the power spectrum of topographic field providing roughness information in the frequency space.

On Mars, different authors have explored the scaling properties of topography by the use of scale invariant parameters. The observed intermittency [4] apparently rejects the idea of a global description of any topographic field at the planetary scale. However, modern developments in the fractal theory might be able to give full account to the observed variability and intermittency. As proposed by [5], it is possible to extent the fractal interpretation of topography to a multifractal statistical object requiring an infinite number of fractal dimensions (one for each statistical moment).

## 2. Data and method

We used the MOLA instrument database to study Mars [6]. The absolute vertical accuracy is ~10 m but depends on accuracy of reconstruction of radial spacecraft orbit. The surface spot size is 130 m. The along-track point spacing is 330 m. The across-track shot spacing depends on mapping orbit and vary with latitude since the orbit is quasi-polar.

The MOLA topography database is available in the PDS archive. This database has been filtered from

noise and atmospheric clouds reflectors. We used the MOLAUtils tools developed to extract all the points in a given surface [7].

The haar fluctuations [8] are computed for all the available along-track points on the MOLA database and splitted in 74 bins of scale from 600m to the planet scale. For each bin of scales, 21 statistical moment are computed (order 0.1 to 2, step 0.1).

### 3. Results

Figure 1 is the main result of this analysis. Although the linear correlation (scaling) is satisfying, two distinct scaling regimes occur with a transition around 10 km. Figure 2 presents the slope of the linear fit for different moments (blue point : <10km, red points : >10km).

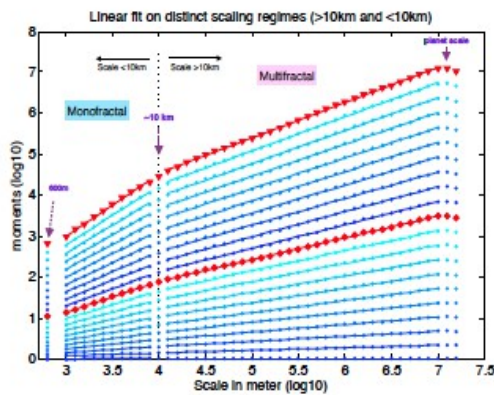


Figure 1: Linear fit on the two different scaling regimes (inferior and superior to 10 km) for every 21 statistical moments from 0.1 to 2.

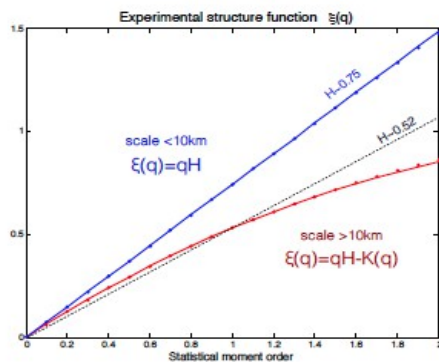


Figure 2: Theoretical structure function  $\zeta(q)$  combining the 21 linear fits shown on figure 1. Red points (resp. blue points) correspond to the range of scales superior (resp. inferior) to 10km

### 4. Discussion and conclusion

Multiscaling seems to occur on a large but restricted range of scale (superior to 10 km) with a Hurst exponent  $H = 0.52$ . At smaller scale, the topography is still scaling but the symmetry is only monofractal with a parameter  $H = 0.75$ .

We demonstrate that a change of processes governing the Martian topography occurs at 10 km. A multiplicative cascade process is occurring at scale higher than 10 km but a simpler monofractal scaling process is occurring a small scale. Craterisation is well known to be a fractal process with a single fractal dimension ([9]). We propose that the low scales are dominated by craterisation processes, at the origin of the monofractal scaling law. This process has already been proposed by [1]. Future investigations are required to understand the multiplicative cascade processes at large scale

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