EPSC Abstracts
Vol. 10, EPSC2015-293, 2015
European Planetary Science Congress 2015

Orbital evolution of viscoelastic bodies: effect of internal structure

M. Káňová, M. Běhounková
Charles University in Prague, Faculty of Mathematics and Physics, Department of Geophysics, Czech Republic
(kanova@karel.troja.mff.cuni.cz)

Abstract

Tidal evolution of planetary orbit and rotation has been traditionally described using specific rheological assumptions: constant phase lag [1] or constant time lag [2]. Such rheologies are, however, unsuitable in the case of terrestrial bodies described by realistic viscosity and rigidity (e.g. [3]), as they predict stable pseudo-synchronous, non-resonant rotation of planets on eccentric orbit, which is not observed in the nature (planetary satellites in the Solar system, planet Mercury). Several authors have recently proposed analytical treatment of orbital evolution with the assumption of a viscoelastic rheology ([3], [4]). Here, we present a numerical approach to the problem and we study the effects of viscosity pattern on the rate of tidal dissipation.

1. Introduction and methods

Unperturbed system of two spherically symmetric bodies maintains constant orbital parameters, determined uniquely by the solution of a two-body problem. The orbital evolution can emerge only when a perturbation is introduced into the system. Such perturbation arises, in our case, as a result of tidal deformation of the planet, breaking off its spherical symmetry.

We therefore first investigate the deformation of a planetary mantle undergoing tidal loading by a host star. The mantle is represented by a viscoelastic Maxwell-like spherical shell and its response to the loading force $\mathbf{f}$ is computed in the time domain as a solution of governing equations

$$\nabla \cdot \mathbf{u} = 0 \,, \tag{1}$$

$$-\nabla \pi + \nabla \cdot \mathbf{D} + \mathbf{f} = \mathbf{0} \,, \tag{2}$$

$$\frac{\partial \mathbf{D}}{\partial t} - \frac{\partial}{\partial t}\left[\mu(\nabla \mathbf{u} + \nabla^{\mathrm{T}} \mathbf{u})\right] = -\frac{\mu}{\eta}\mathbf{D} \,, \tag{3}$$

where $\mathbf{u}$ is the displacement vector, π and $\mathbf{D}$ represent the isotropic and the deviatoric part of the stress tensor, μ is the effective shear modulus and η the effective viscosity [5]. The force $\mathbf{f}$ consist of two contributions: the first one due to the external potential and the second one due to the self-gravity of a deformed planet. We solve the equations using a spherical harmonic decomposition in the lateral direction and a staggered finite difference method in the radial direction [6].

The mass excess or deficit due to the boundary deflections of a deformed shell enables us to compute the disturbation of the external field. The disturbing force $\mathbf{f}_{\mathrm{dist}}$ is evaluated in the instantaneous position of a host star and decomposed into three orthogonal components: $\mathbf{R}$ in the direction of radius vector, $\mathbf{S}$ perpendicular to $\mathbf{R}$ in the orbital plane, pointing in the direction of planetary motion, and $\mathbf{W}$ perpendicular to the orbital plane, pointing in the direction of orbital angular momentum. Perturbation of the planetary orbit for a system containing one star and one planet is then computed using Gauss planetary equations:

$$\frac{\mathrm{da}}{\mathrm{dt}} = \frac{2}{n\sqrt{1-e^2}}\left[eR\sin\nu + \frac{p}{r}S\right] \,, \tag{4}$$

$$\frac{\mathrm{de}}{\mathrm{dt}} = \frac{\sqrt{1-e^2}}{nae}\left[eR\sin\nu + \left(\frac{p}{r} - \frac{r}{a}\right)S\right] \,. \tag{5}$$

Here a and e symbolize the semi-major axis and the eccentricity, respectively, n is the mean motion of the planet, ν the true anomaly, r represents the instantaneous distance of the planet from the star, $p = a(1-e^2)$ is the semi-latus rectum and R, S are the magnitudes of the first two components of the disturbing (tidal) force.

Once we know the average values of secular change in a and e, we compute the long-term orbital evolution explicitly. The rotational period Ω of a spherical planet with the moment of inertia C evolves in agreement with the conservation of total angular momentum of the system.

$$\frac{Mm}{M+m}\, a^2 n \sqrt{1-e^2} \; + \; C\Omega \; = \; \text{const.} \qquad (6)$$

2. Spin-orbit resonances

Study of the long-term evolution of any planetary processes depending on the surface temperature and/or tidal loading requires realistic model for the orbital evolution. Locking of a planet into a spin-orbit resonance results in insolation and temperature pattern that is qualitatively different from that of pseudo-synchronous state.

Figure 1 shows stable spin-orbit ratios for a close-in terrestrial planet with various values of eccentricity and effective viscosity (note that the effective viscosity for tidal deformation is lower than the viscosity used in mantle convection models—the mean effective viscosity of the Earth mantle would be $\sim 10^{18}\,\mathrm{Pa.s}$). The results demonstrate that while for some values of viscosity η and Maxwell time $\tau_M = \frac{\eta}{\mu}$ the equillibrium rotation state is well described by the constant time lag model (purely viscous or elastic limit), other Maxwell bodies tend to get locked into spin-orbit resonance. The upper left plot of Figure 1 depicts an intermediate state between discrete spin-orbit resonances and pseudo-synchronous rotation, a result for a viscoelastic planet with very low viscosity.

3. Summary and Conclusions

We have implemented a numerical model for the orbital and rotational evolution of viscoelastic planets and studied parameter dependence of tidal dissipation and stable spin-orbit ratios, including resonances. The model enables evaluation of effects of the internal viscosity structure of the planet and may be useful in further studies of coupled internal and orbital evolution.

Acknowledgements

This work was supported by the Grant Agency of Charles University (project No. 338214) and the Czech Science Foundation (project No. 14-04145S).

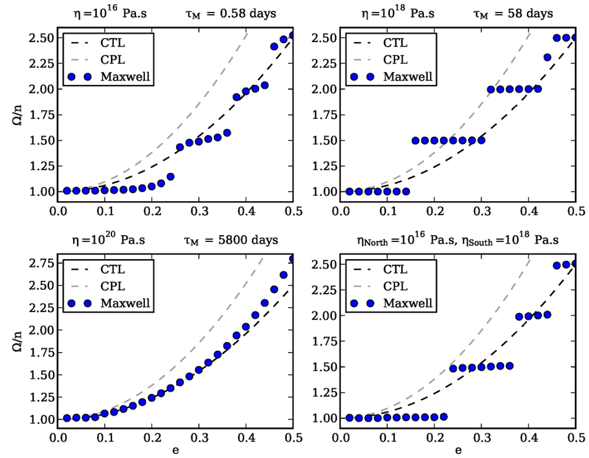

Figure 1: Stable spin-orbit ratios as a function of eccentricity for different values of effective viscosity. Comparison with traditional models: CTL = constant time lag, CPL = constant phase lag. Earth-like planet orbiting Sun-like star, $a = 0.055\,\mathrm{AU}$, $T_{orb} = 4.71\,\mathrm{days}$, $\mu = 2.10^{11}\,\mathrm{Pa}$.

References

[1] Kaula, W. M.: *Tidal Dissipation by Solid Friction and the Resulting Orbital Evolution*, Reviews of Geophysics 2, pp. 661-685, 1964.

[2] Mignard, F.: *The evolution of the lunar orbit revisited*, The Moon and the Planets 20, pp. 301-315, 1979.

[3] Makarov, V. V., Efroimsky, M.: *No pseudosynchronous rotation for terrestrial planets and moons*, The Astrophysical Journal 764:27, 2013.

[4] Correia, A. C. M., Boué, G., Laskar, J., Rodríguez, A.: *Deformation and tidal evolution of close-in planets and satellites using a Maxwell viscoelastic rheology*, Astronomy & Astrophysics 571, A50, 2014.

[5] Běhounková, M., Tobie, G., Choblet, G., Čadek, O.: *Coupling mantle convection and tidal dissipation: Applications to Enceladus and Earth-like planets*, Journal of Geophysical Research 115, E09011, 2010.

[6] Tobie, G., Čadek, O., Sotin, C.: *Solid tidal friction above a liquid water reservoir as the origin of the south pole hotspot on Enceladus*, Icarus 196, pp. 642–652, 2008.