

# Realistic uncertainties on Hapke model parameters from photometric measurements

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## Abstract

We propagate the uncertainties from the photometric measurement to the Hapke's photometric parameter using the Bayesian Monte Carlo approach. Since non-linearities are strong, uncertainties may have a non-Gaussian shape, especially in the case of relatively large uncertainties. We propose here to study synthetic examples in order to characterize the uncertainties of previous analysis but also to propose new strategies for new acquisition campaigns.

## 1. Introduction

The single particle phase function describes the manner in which an average element of a granular material diffuses the light in the angular space. Interestingly, this function is related to the particle properties, such surface roughness or internal scatterers. Usually, this function is approximated by the Henyey-Greenstein function with two parameters: the asymmetry parameter  $b$  describing the width of the scattering lobe and the backscattering fraction  $c$  describing the main direction of the scattering lobe. Hapke proposed a convenient analytical model to describe the spectro-photometry of granular materials [1]. This model has been widely used to interpret telescopic observations, remote sensing data, in situ and laboratory measurements [2,3]. Using a compilation of the published data, Hapke [4] recently studied the relationship of  $b$  and  $c$  for natural examples and proposed the hockey stick relation. For the moment, there is no theoretical explanation for this relationship. One goal of this project is to study a possible bias due to the retrieval method.

## 2. Method

We expand here an innovative Bayesian inversion method in order to study into detail the uncertainties of retrieved parameters (single scattering albedo, surface roughness,  $b$ ,  $c$ , opposition effect) [5,6,7].

Indeed, the main advantage of this approach is that it provides all the possible solutions over the parameter domain through a probability density function (PDF). We performed sensitivity tests by mimicking various surface scattering properties and various single image-like/disk resolved image, Emission Phase Function (EPF)-like and Bidirectional Reflectance Distribution Function (BRDF)-like geometric sampling conditions under varied geometric conditions. Moreover, we estimated the favorable geometric conditions for an accurate estimation of photometric parameters in order to provide new constraints for future observation campaigns and instrumentations [8].

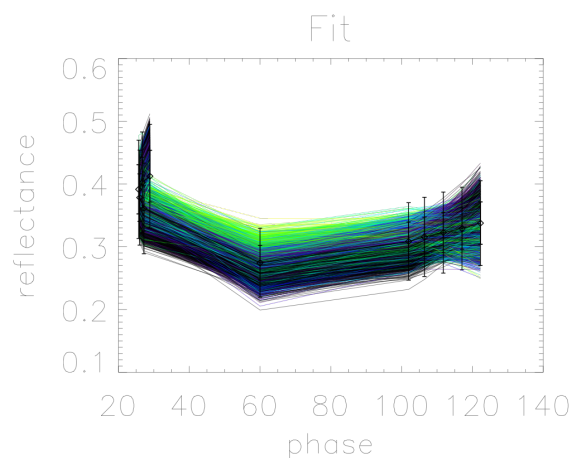


Figure 1: Example of a result on a synthetic observation with 10% uncertainty. The black curve represents the synthetic data with one and two standard deviations. Light color curves represent 500 sampled solutions from the Monte Carlo Markov Chain.

## 3. Results

As an example, we present in figures 1 to 3, the results for an estimation of the retrieved uncertainties in the following conditions: incidence= $60^\circ$  along the

azimuthal plane= $\{30^\circ; 210^\circ\}$  resulting to a phase angle range from  $29^\circ$  to  $122^\circ$ , and using the following model parameter set:  $\omega=0.9$ ,  $b=0.8$ ,  $c=0.1$ ,  $\theta = 15^\circ$ ,  $B_0=1$  and  $h=0$ . The uncertainty is set to 10% in REFF unit and independent for each geometry.

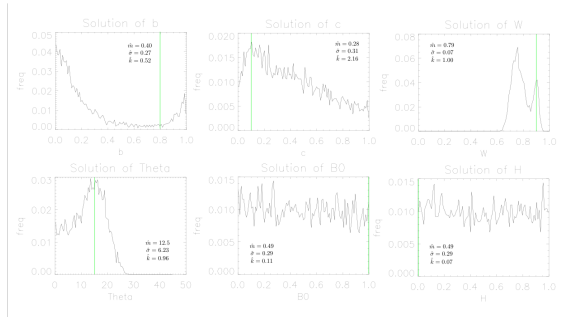


Figure 2: PDF of the solution, for each parameter of the Hapke's parameter on the same example as fig. 1. Each plot represents the histogram of the 500 solutions from the Monte Carlo Markov Chain. The color line represents the initial parameters set.

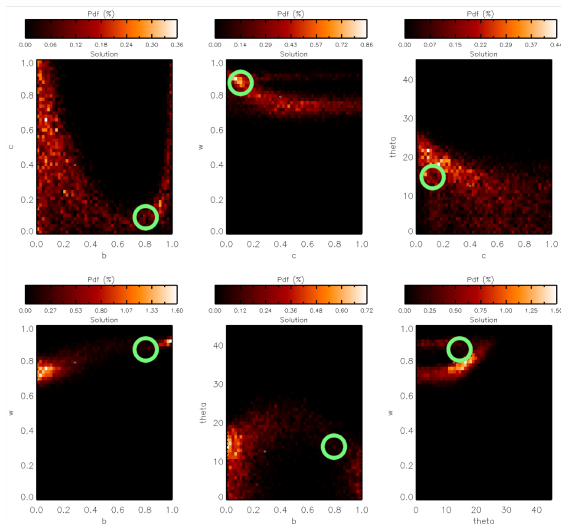


Figure 3: Probability Density Function (PDF) of the solution, for each couple of constrained parameters ( $\omega$ ,  $b$ ,  $c$ ,  $\theta$ ) on the same example as fig. 1 and 2. The black/white diamonds represent the average of the PDF. The green circles represent the expected values each parameter.

## 4. Summary and Conclusions

Some conclusions can be addressed [8]:

Non-linearities in the Hapke model are important leading to potential multiple solutions for EPF type measurements with at least data uncertainties larger than 5% and large azimuthal plane angle ( $> 30^\circ$ ).

One single EPF type observation with very favorable conditions (i.e., principal plane, incidence at  $75^\circ$ , emergence angle sampling up to  $80^\circ$ ) is enough to constrain the parameters, even with data uncertainty level of 10%.

For data uncertainty less than 5% (for common spaceborne and laboratory measurements), the parameters can be estimated using single EPF, even with large uncertainties, under certain geometric configurations: close to the principal plane (azimuthal angle less than  $45^\circ$ ) and high incidence angles (greater than  $50^\circ$ ) leading to a broad phase angle range containing low and high phase angles to sufficiently describe the shape of the photometric curves).

On EPF data, we demonstrate that the uncertainties of the retrieved parameters follow the same hockey stick relation, suggesting that this relation is due to the fact that  $b$  and  $c$  are coupled parameters in the Hapke's model instead of natural phenomena. Nevertheless, the data used in the Hapke (2012) compilation generally are full BRDF which are not to be subject to this artifact.

## References

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