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Numerical approach to constructing the lunar physical libration: results of the initial stage

Zagidullin A. (1), Petrova N. (1,2), Usanin V. (1), Nefedief Yu. (1), Glushkov.M (1)

(1) Kazan Federal University, Kazan, Kremlevskya str., 18, Russia (arhtur.zagidullin@ya.ru)
(2) Kazan Power Engineering University, Kazan, Russia

Abstract

So called "main problem" it is taken as a model to develop the numerical approach in the theory of lunar physical libration. For the chosen model, there are both a good methodological basis and results obtained at the Kazan University as an outcome of the analytic theory construction. Results of the first stage in numerical approach are presented in this report.

Three main limitation are taken to describe the main problem:

- independent consideration of orbital and rotational motion of the Moon;
- a rigid body model for the lunar body is taken and its dynamical figure is described by inertia ellipsoid, which gives us the mass distribution inside the Moon.
- only gravitational interaction with the Earth and the Sun is considered. Development of selenopotential is limited on this stage by the second harmonic only. Inclusion of the 3-rd and 4-th order harmonics is the nearest task for the next stage.

The full solution of libration problem consists of removing the below specified limitations: consideration of the fine effects, caused by planet perturbations, by visco-elastic properties of the lunar body, by the presence of a two-layer lunar core, by the Earth obliquity, by ecliptic rotation, if it is taken as a reference plane.

1. Mathematical Formulation of the problem

The technique of mathematical description of the lunar rotation and solution obtaining was developed by Khabibullin [1] and later by Petrova [2]. The rotation of the Moon as a rigid body is described by a trihedron of its principle inertia axes, which are rigidly bounded with the lunar body. This trihedron, whose origin from the lunar center of mass, is called Dynamical Coordinate System (DCS). The motion of the lunar body is described by the angles μ, ν, π (Fig. 1).

The kinematical equations

$$\Omega_x = -\dot{\mu} \times \sin\nu - \dot{\pi}$$
$$\Omega_y = -\dot{\mu} \times \cos\nu \times \sin\pi + \dot{\pi} \times \cos\pi$$
$$\Omega_z = \dot{\mu} \times \cos\nu \times \cos\pi + \dot{\pi} \times \sin\pi$$

give us projections of the rotation angular velocity Ω on the reference axes, which are taken from ecliptical coordinate system.

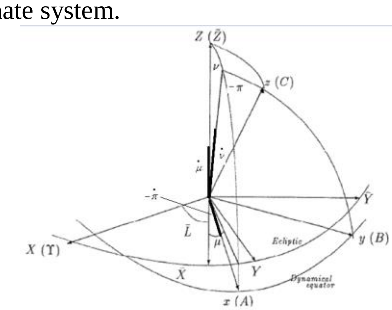

Fig 1: To the conclusion of the kinematic equations

The equations of the lunar rotation are being built in Hamilton formalism, where the canonical variables are $q_1 = \mu, q_2 = \nu, q_3 = \pi$ and the conjugate canonical momenta are p_1, p_2, p_3. Initial values for the numerical integration are calculated according to the analytical theory [2]. The same theory has been used as a reference to verify the accuracy of the numerical integrator.

The equations obtained were greatly simplified to accelerate the debugging programs for the numerical integration: trigonometric functions were expanded into series on small variables (q_1, q_2, q_3) till the terms of the first order of smallness $O(q_1^2, q_2^2, q_3^2)$. In fact, the resulting system of equations

$$\dot{q}_1 = p$$
$$\dot{q}_2 = (1+\kappa_1)p_2 - \kappa_1 n q_3$$
$$\dot{q}_3 = (1+\kappa_2)p_3 - n q_2$$
$$\dot{p}_1 = Q_{100}(t)$$
$$\dot{p}_2 = -n^2 q_2 + n p_3 + Q_{010}(t)$$
$$\dot{p}_3 = -\kappa_1 n^2 q_3 + \kappa_1 n p_2 + Q_{001}(t)$$

corresponds to a linear problem of physical libration. Here the function Qijk (t) are the trigonometric series coming from expansion of selenopotential till the 2nd order. The advantage of the system (2) is that it allows obtaining exact analytical solutions. This system was solved numerically.

2. Numerical Integration of Lunar libration equation

The one-step Runge Kutta (RK) method of 4^{th} (RK-4) and 10^{th} (RK-10) orders of accuracy [3] was used to numerical integration.

On the Fig. 2a, 2b we can see the behavior of error in dependence on a step in the interval of 1020 days. The optimal step for RK-10 is ½ day. Fig. 3a, 3b demonstrate results of calculation in the 10 years interval: that is difference between numerical and exact solutions.

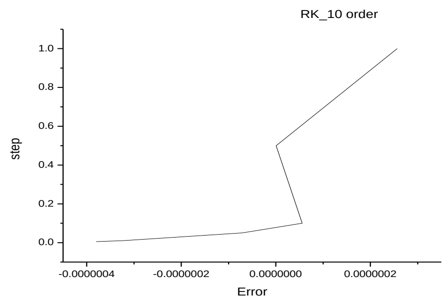

Fig 2a: Error for RK 10 order

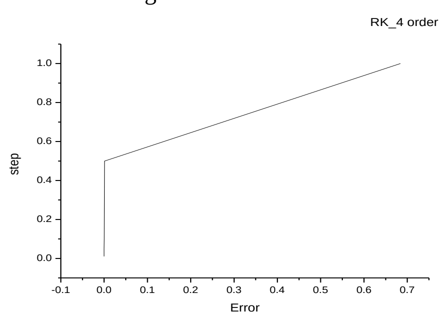

Fig 2b: Error for RK 4 order

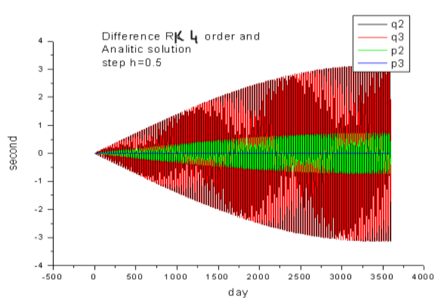

Fig 3a: Error in the interval of 10 years for RK-40.

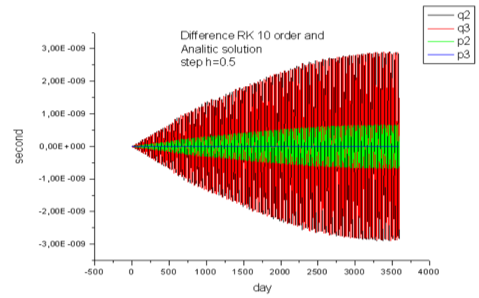

Fig 3b: Error in the interval of 10 years for RK-10.

Analyzing of the curves leads to the conclusion, that the RK-10 is on 4 order better than RK-4. Decreasing of a step in the method of RK-4 leads to the accuracy of the method of RK-10, but the calculation time increases in several times. Because of this we choose the RK-10 as main integrator. The 1/2 day is a good step for the physical libration, because there are no so short harmonics in lunar libration.

3. Summary and Conclusions

The modern accuracy of the lunar laser ranging has an accuracy of 0.0001 arc seconds for lunar physical libration parameters [4]. The high accuracy is necessary and for other space experiments, such as the planned Japanese project ILOM (In situ Lunar Orientation Measurement), which is directed to obtain millisecond accuracy in determination of the lunar libration [5].

Integrator RK-4 shows a stable error ±5 seconds at the 10-years interval, while the RK-10 has accuracy 10^{-9} second and an acceptable calculation time at the same time interval. We believe this method will justify itself in solving more complex, actual, equations of libration.

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